Online Appendix for "What Do We Learn From Cross-Regional Empirical Estimates in Macroeconomics?"

Adam Guren — Boston University
Alisdair McKay — FRB of Minneapolis
Emi Nakamura — UC Berkeley
Jón Steinsson — UC Berkeley

May 12, 2020

C Partial equilibrium housing wealth effect

C.1 Household's problem and consumption function

The household maximizes

$$\sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t, H_t\right),$$

where C_t, N_t, H_t refer to consumption, labor supply and housing, respectively. The budget constraint is:

$$p_t H_t + C_t + B_t = W_t N_t + D_t + R_{t-1} B_{t-1} + p_t H_{t-1}(1-\delta),$$

where R_t is the gross real interest rate between t and $t+1$ and we have abstracted from the portfolio holding cost. The first order conditions are:

$$\begin{aligned}
u_{C,t} &= \lambda_t \\
u_{H,t} &= \lambda_t p_t - \beta \lambda_{t+1} p_{t+1}(1-\delta) \\
\lambda_t &= \beta R_t \lambda_{t+1} \\
u_{N,t} &= -\lambda_t W_t
\end{aligned}$$

Combining we have:

$$
\begin{aligned}
u_{C,t} &= \beta R_t u_{C,t+1} \\
\frac{u_{H,t}}{u_{C,t}} &= p_t - R_t^{-1} p_{t+1}(1-\delta) \\
-\frac{u_{N,t}}{u_{C,t}} &= W_t.
\end{aligned}
$$

Using the period utility function and rearranging yields:

$$
\begin{aligned}
N_t &= \left(\frac{W_t}{\psi}\right)^{1/\nu} \\
\tilde{C}_t &\equiv C_t - \psi \frac{N_t^{1+\nu}}{1+\nu} \\
\tilde{H}_t &\equiv H_t - \Omega_t \\
x_t &\equiv \frac{1-\kappa}{\kappa} \left[p_t - R_t^{-1} p_{t+1}(1-\delta)\right]^{-1} \\
\tilde{H}_t &= x_t \tilde{C}_t \\
u_{C,t} &= \kappa x_t^{(1-\kappa)(1-\sigma)} \tilde{C}_t^{-\sigma} \\
\tilde{C}_{t+1} &= \left[\beta R_t \left(\frac{x_{t+1}}{x_t}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} \tilde{C}_t \\
\tilde{C}_t &= X_{0,t} \tilde{C}_0,
\end{aligned}
$$

where:

$$
\begin{aligned}
X_{0,t} &= \prod_{s=1}^{t} \left[\beta R_{s-1} \left(\frac{x_s}{x_{s-1}}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} \\
X_{0,t} &= \left[\beta^t R_{0,t} \left(\frac{x_t}{x_0}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma}.
\end{aligned}
$$

Now using the present value budget constraint:

$$
\sum_{t=0}^{\infty} R_{0,t}^{-1} \left[p_t \left(H_t - (1-\delta) H_{t-1}\right) + C_t - W_t N_t - D_t\right] = R_{-1} B_{-1},
$$

substituting in for C_t and H_t and rearranging gives:

$$\tilde{C}_0 = \kappa \frac{p_0(1-\delta)\tilde{H}_{-1} + R_{-1}B_{-1} + \sum_{t=0}^{\infty} R_{0,t}^{-1}\left[W_t N_t + D_t - \psi \frac{N_t^{1+\nu}}{1+\nu} - \Omega_t \frac{1-\kappa}{\kappa x_t}\right]}{\sum_{t=0}^{\infty} R_{0,t}^{-1} X_{0,t}}. \tag{1}$$

C.2 Complete markets

To derive equation (47) we use steps that are similar to the incomplete markets case in section C.1. The date-0 budget constraint is:

$$\sum_{t=0}\sum_{s^t} \pi_t(s^t)\Xi_{0,t}(s^t)\left[C_t(s^t) + p_t(s^t)H_t(s^t) - p_t(s^t)H_{t-1}(s^{t-1})(1-\delta)\right] = \text{ initial wealth}$$

where s^t is a history up to date t and $\Xi_{0,t}(s^t)$ is the date-0 price for the Arrow-Debreu security that pays off in that history. The FOCs for C_t and H_t are

$$\begin{aligned}
\kappa \tilde{C}_t^{\kappa(1-\sigma)-1}\tilde{H}_t^{(1-\kappa)(1-\sigma)} &= \lambda \Xi_{0,t} \\
(1-\kappa)\tilde{C}_t^{\kappa(1-\sigma)}\tilde{H}_t^{(1-\kappa)(1-\sigma)-1} &= \lambda \Xi_{0,t}\left[p_t - \mathbb{E}_t\left[q_{t+1}p_{t+1}(1-\delta)\right]\right]
\end{aligned}$$

where λ is the multiplier of the budget constraint, $q_{t+1} \equiv \Xi_{0,t+1}/\Xi_{0,t}$ and we are using the notation $\tilde{C}$ and $\tilde{H}$ introduced in section C.1. Combining these yields

$$\tilde{H}_t = \frac{1-\kappa}{\kappa}\left[p_t - \mathbb{E}_t\left[q_{t+1}p_{t+1}(1-\delta)\right]\right]^{-1}\tilde{C}_t$$

and substituting into the FOC for C yields

$$\kappa \tilde{C}_t^{-\sigma}\left(\frac{1-\kappa}{\kappa}\left[p_t - \mathbb{E}_t\left[q_{t+1}p_{t+1}(1-\delta)\right]\right]^{-1}\right)^{(1-\kappa)(1-\sigma)} = \lambda \Xi_{0,t}.$$

Assuming equal initial wealth, the two regions will have the same Lagrange multiplier on the date-0 budget constraint so the right-hand side of the above equation will be the same in the home region and foreign region. Equating the left-hand sides and rearranging yields equation (47).

C.3 Computing the partial equilibrium housing wealth effect

We now explain how we compute the partial equilibrium consumption response to a change in home prices. The complication comes from the fact that consumption at t depends on expectations of all future home prices. As the details of the expectations matter, we want to make sure we are using the same expected path for home prices in this calculation as the one that arises in the simulation of the full GE model. In this subsection we describe how we do that using the VAR representation of the GE economy denoted by the state vector of the economy $\mathcal{X}_t$ and matrices $\mathcal{P}$ and $\mathcal{Q}$ such that $\mathcal{X}_t = \mathcal{P}\mathcal{X}_{t-1} + \mathcal{Q}\epsilon_t$.

As we are focusing on partial equilibrium fluctuations in home prices, the sum in the numerator of (1) is constant. The sum in the denominator depends on all future home prices. To a first order approximation around a steady state with $\beta R = 1$, this sum can be written as:

$$\sum_{t=0}^{\infty} R_{0,t}^{-1}\left[\beta^t R_{0,t}\left(\frac{x_t}{x_0}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} = \sum_{t=0}^{\infty} R^{-t}\left(\frac{Rp_0 - p_1(1-\delta)}{Rp_t - p_{t+1}(1-\delta)}\right)^{\frac{(1-\kappa)(1-\sigma)}{\sigma}}$$
$$\approx \frac{(1-\kappa)(1-\sigma)}{\sigma}\frac{R}{(R-1+\delta)\bar{p}}\left[\frac{R}{R-1}\left(p_0 - \frac{1-\delta}{R}p_1\right) - \sum_{t=0}^{\infty}\bar{R}^{-t}\left(p_t - \frac{1-\delta}{R}p_{t+1}\right)\right]. \quad (2)$$

To compute this recursively, note that $\mathbb{E}\left[\mathcal{X}_t\right] = \mathcal{P}^t\mathcal{X}_0$. Moreover, using $\mathcal{I}$ as a column vector that gives the linear mapping from $\mathcal{X}_t$ to p_t we have:

$$\begin{aligned}\mathbb{E}\sum_{t=0}^{\infty} R^{-t}p_t &= \mathbb{E}\sum_{t=0}^{\infty} R^{-t}\mathcal{I}\mathcal{X}_t \\ &= \mathcal{I}\sum_{t=0}^{\infty} R^{-t}\mathcal{P}^t\mathcal{X}_0 \\ &= \mathcal{I}\left(I - R^{-1}\mathcal{P}\right)^{-1}\mathcal{X}_0.\end{aligned}$$

Using this, equation (2) becomes:

$$\approx \frac{(1-\kappa)(1-\sigma)}{\sigma}\frac{R}{(R-1+\delta)\bar{p}}\mathcal{I}\left[\frac{R}{R-1}\left(I - \frac{1-\delta}{R}\mathcal{P}\right) - \left(I - R^{-1}\mathcal{P}\right)^{-1}\left(I - \frac{1-\delta}{R}\mathcal{P}\right)\right]\mathcal{X}_0. \quad (3)$$

This approach can be applied to the regime switching model by expanding the state vector so that

$$\tilde{\mathcal{X}}_0 = \begin{bmatrix} \mathcal{X}_t \\ \mathbf{0} \end{bmatrix}$$

and the state transition matrix is

$$\tilde{\mathcal{P}} = \left[\begin{array}{cc} (1-\omega)P_{\text{short}} & 0 \\ \omega P_{\text{long}} & P_{\text{long}} \end{array} \right],$$

where ω is the regime-switching probability, P_{short} is the state transition matrix when staying in the short-run regime and P_{long} is the state transition matrix in the long-run regime. One would also set

$$\tilde{\mathcal{I}} = \left[\begin{array}{cc} \mathcal{I}_{\text{short}} & \mathcal{I}_{\text{long}} \end{array} \right].$$

In calculating the partial equilibrium housing wealth effect, we simulate the general equilibrium model in response to aggregate housing demand shocks and at each date in the simulation we record $\mathcal{X}_t$, H_{t-1}, B_{t-1} and p_t. We then plug these values into (1) with the denominator computed by (3). This gives us a time-series of "partial equilibrium" consumption for each region to go along with the time series of home prices. We then regress the difference in log consumption across regions on the difference in log home prices. Notice that the partial equilibrium consumption series will vary over time with the bond positions the two regions have inherited from the past. In the absence of a portfolio holding cost, these bond positions are non-stationary and the partial equilibrium housing wealth effect becomes unstable.

D Additional Derivations

D.1 Intermediate Good's Price-Setting

Substituting the demand curve into the objective function yields

$$\max_{\breve{\mathcal{P}}_t} \mathbb{E}_t \sum_{\tau=t}^{\infty} \chi^t \lambda_{t,\tau} Y_\tau \left(\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_\tau} \left(\frac{\breve{\mathcal{P}}_t}{\mathcal{P}_{H,\tau}} \right)^{1-\eta} - w_\tau \left(\frac{\breve{\mathcal{P}}_t}{\mathcal{P}_{H,\tau}} \right)^{-\eta} \right)$$

and the first order condition is

$$\frac{\breve{\mathcal{P}}_t}{\mathcal{P}_{H,t}} = \frac{\eta}{\eta-1} \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} \chi^t \lambda_{t,\tau} Y_\tau w_\tau \left(\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_{H,t}} \right)^{\eta}}{\mathbb{E}_t \sum_{\tau=t}^{\infty} \chi^t \lambda_{t,\tau} Y_\tau \frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_\tau} \left(\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_{H,t}} \right)^{\eta-1}}.$$

Observe that the ratio $\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_\tau}$ in the denominator can be re-expressed using the price index for the domestic consumption bundle $\mathcal{P}_\tau = \mathcal{P}_{H,\tau}^{\phi} \mathcal{P}_{F,\tau}^{1-\phi}$ as $\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_\tau} = \left(\frac{\mathcal{P}_{H,\tau}}{\mathcal{P}_{F,\tau}} \right)^{1-\phi}$.

Turning to inflation dynamics, define $\pi_{H,t} \equiv \mathcal{P}_{H,t}/\mathcal{P}_{H,t-1}$, define $\pi_{F,t}$ analogously and define $\pi_t \equiv \mathcal{P}_t/\mathcal{P}_{t-1}$ as the inflation rate of the price index associated with the domestic consumption bundle (CPI). These inflation rates are determined according to

$$\pi_{H,t} = \left(\theta^{-1} - \frac{1-\theta}{\theta}\left(\frac{\breve{\mathcal{P}}}{\mathcal{P}_{H,t}}\right)^{1-\eta}\right)^{1/(\eta-1)}$$

$$\pi_t = \frac{\mathcal{P}_{H,t}^{\phi}\mathcal{P}_{F,t}^{1-\phi}}{\mathcal{P}_{H,t-1}^{\phi}\mathcal{P}_{F,t-1}^{1-\phi}} = \pi_{H,t}^{\phi}\pi_{F,t}^{1-\phi}$$

and analogous equations for the foreign region.

E Supply Curve Heterogeneity and the Estimated Investment Response to Home Prices

Figure 1 shows a negative relationship between home prices and residential investment. This appendix describes this relationship in econometric terms. The appendix then shows that applying our adjustment formula from Section 3 still yields the correct partial-equilibrium housing wealth effect despite the complications from heterogeneity in housing supply curves.

An estimate of the housing wealth effect might regress the change in consumption on the change in home prices and a constant or time fixed effect. In this discussion, we will work with the equations of the full model, but we will assume that the dynamic relationships between variables are dominated by the static relationships so the matrices $\mathbf{C}_Y$ and $\mathbf{C}_p$ are (approximately) diagonal.[1]

The regression specification is easier to describe in terms of demeaned variables rather than cross-region differences. Using similar steps as in the previous subsection yields:

$$Y_r - \bar{Y} = \Phi\left(C_r - \bar{C} + I_r - \bar{I}\right) + G_r - \bar{G},$$

where Y_r is income in region r and $\bar{Y}$ is population-weighted average income across regions. As above $\Phi \equiv \phi + \phi^* - 1$. Using the linearized consumption function and the equation above we can write:

$$C_r - \bar{C} = \mathbf{M}\mathbf{C}_p\left(p_r - \bar{p}\right) + \mathbf{C}_Y\Phi\left(I_r - \bar{I}\right) + \mathbf{C}_Y\left(G_r - \bar{G}\right). \tag{4}$$

[1] A diagonal $\mathbf{C}_Y$ implies $\mathbf{M}$ is diagonal. $\mathbf{I}_p$ is already diagonal as residential investment only depends on the current home price (see equation 35).

In the model, residential investment is increasing in home prices, but with a different slope in each region due to heterogeneous housing supply elasticities. We can write:

$$I_r - \bar{I} = \mathbf{I}_{r,p} p_r - \mathbf{I}_p \bar{p} = \mathbf{I}_p (p_r - \bar{p}) + (\mathbf{I}_{r,p} - \mathbf{I}_p) p_r,$$

where $\mathbf{I}_{r,p}$ is the slope of the residential investment response to home prices in region r and $\mathbf{I}_p$ is defined so that $\bar{I} = \mathbf{I}_p \bar{p}$. Equation (4) can then be written as:

$$C_r = \underbrace{(\mathbf{M}\mathbf{C}_p + \mathbf{C}_Y \Phi \mathbf{I}_p)}_{\text{coef. of interest}} (p_r - \bar{p}) + \underbrace{\mathbf{C}_Y \Phi \left(\mathbf{I}_{r,p} - \mathbf{I}_p\right) p_r + \mathbf{C}_Y \left(G_r - \bar{G}\right)}_{\text{error}} + \underbrace{\bar{C}}_{\text{time fixed effect}} . \quad (5)$$

Changes in aggregate variables (i, Ω, T) affect all regions equally and are absorbed by the time fixed effect. The response of residential investment to home prices can be written:

$$I_r = \underbrace{\bar{\mathbf{I}}_p}_{\text{coef. of interest}} (p_r - \bar{p}) + \underbrace{\left(\mathbf{I}_{r,p} - \bar{\mathbf{I}}_{r,p}\right) p_r}_{\text{error}} + \underbrace{\bar{I}}_{\text{time fixed effect}} . \quad (6)$$

Equations (5) and (6) show a potential source of bias in the housing wealth effect regression: To the extent that cities differ in their housing supply elasticities they will differ in the response of residential investment to home prices and cities with larger price changes will have smaller elasticities of residential investment. The treatment effects are heterogeneous and the treatment (price changes) are negatively correlated with the treatment effect (cities with less responsive residential investment have larger price changes). Therefore the estimated average treatment effect is not the population average effect.[2] This bias affects both the measured housing wealth effect and the construction regressions.

A benefit of the adjustment we put forward here is that the bias in the two regressions cancels out when we compute the partial equilibrium housing wealth effect. To see this, when we estimate equation (5) we obtain a coefficient of interest of (see Appendix E.1):

$$\breve{\gamma}^C = \bar{\gamma}^C + \frac{\mathbb{E}\left[\left(\gamma_r^C - \bar{\gamma}^C\right) p_{r,t} \tilde{p}_{r,t}\right]}{\mathbb{E}\left[\tilde{p}_{r,t}^2\right]},$$

where $\gamma_r^C \equiv \mathbf{M}\left(\mathbf{C}_p + \mathbf{C}_Y \Phi \mathbf{I}_{r,p}\right)$, $\tilde{p}_{r,t} \equiv p_{r,t} - \bar{p}_t$, and $\breve{\gamma}$ is the estimated value of γ. When we estimate

[2] IV strategies that use supply constraints as instruments for home prices will not overcome this bias because the price variation they isolate is still correlated with the treatment effects.

(6) we obtain:

$$\breve{\gamma}^I = \bar{\gamma}^I + \frac{\mathbb{E}\left[\left(\gamma_r^I - \bar{\gamma}^I\right) p_{r,t}\tilde{p}_{r,t}\right]}{\mathbb{E}\left[\tilde{p}_{r,t}^2\right]},$$

where $\gamma_r^I \equiv \mathbf{I}_{r,p}$. Crucially, note that the regional variation in γ_r^C comes only from $\mathbf{I}_{r,p}$ so we have $\gamma_r^C - \bar{\gamma}^C = \mathbf{M}\mathbf{C}_Y\Phi\left(\gamma_r^I - \bar{\gamma}^I\right)$.

To put the pieces together, we form $dE/dp \equiv \mathbf{M}\left(\mathbf{C}_p + \mathbf{I}_p\right)$ by summing the coefficients of interest in in (5) and (6).[3] This gives:

$$\frac{d\breve{E}}{dp} = \bar{\gamma}^C + \bar{\gamma}^I + \mathbf{M}\frac{\mathbb{E}\left[\left(\gamma_r^I - \bar{\gamma}^I\right) p_{r,t}\tilde{p}_{r,t}\right]}{\mathbb{E}\left[\tilde{p}_{r,t}^2\right]}.$$

Now applying our adjustment:

$$\begin{aligned}
\breve{\mathbf{C}}_p &= \frac{\frac{d\breve{E}}{dp}}{\mathbf{M}} - \breve{\gamma}^I \\
&= \frac{\bar{\gamma}^C + \bar{\gamma}^I}{\mathbf{M}} + \frac{\mathbb{E}\left[\left(\gamma_r^I - \bar{\gamma}^I\right) p_{r,t}\tilde{p}_{r,t}\right]}{\mathbb{E}\left[\tilde{p}_{r,t}^2\right]} - \bar{\gamma}^I - \frac{\mathbb{E}\left[\left(\gamma_r^I - \bar{\gamma}^I\right) p_{r,t}\tilde{p}_{r,t}\right]}{\mathbb{E}\left[\tilde{p}_{r,t}^2\right]} \\
&= \frac{\mathbf{M}\left(\mathbf{C}_p + \mathbf{C}_Y\Phi\bar{\mathbf{I}}_{r,p}\right) + \bar{\mathbf{I}}_{r,p}}{\mathbf{M}} - \bar{\mathbf{I}}_{r,p} \\
&= \mathbf{C}_p,
\end{aligned}$$

where $\bar{\mathbf{I}}_{r,p}$ is the average $\mathbf{I}_{r,p}$ over r. In the second line the bias to the housing wealth effect on expenditure cancels with the bias in the residential investment response. Underlying this result is the fact that the heterogeneity in treatment effects in the two regressions has the same underlying source (the heterogeneity in housing supply curves). When we remove the estimated residential investment response from the estimated housing wealth effect we end up removing the bias.

E.1 Bias in Estimating Equations (5) and (6)

Consider the data generating process:

$$y_{r,t} = f_t + \gamma_r p_{r,t} + \varepsilon_{r,t},$$

where t indexes time and r regions. Note that each region has its own γ_r. However, when we estimate the housing wealth effect we estimate a single γ. We do not recover the average γ across

[3] Recall $\mathbf{M} = 1 + \mathbf{C}_Y\Phi\mathbf{M}$.

regions if, say, regions with larger γ's tend to have smaller fluctuations in home prices $p_{r,t}$, which is the implication of regions with more elastic housing supply having residential investment respond more to home prices but home prices fluctuate less.

We estimate with demeaned variables to eliminate f_t. Let $\tilde{y}_{r,t} = y_{r,t} - \bar{y}_t$. We then have:

$$\begin{aligned}
\tilde{y}_{r,t} &= \gamma_r p_{r,t} - \overline{\gamma p}_t + \tilde{\varepsilon}_{r,t} \\
\tilde{y}_{r,t} &= \bar{\gamma} p_{r,t} - \bar{\gamma} J_{r,t} + \gamma_r p_{r,t} - \bar{\gamma}\bar{p}_t - cov_t + \tilde{\varepsilon}_{r,t} \\
\tilde{y}_{r,t} &= \bar{\gamma}\tilde{p}_{r,t} + (\gamma_r - \bar{\gamma})\, p_{r,t} - cov_t + \tilde{\varepsilon}_{r,t},
\end{aligned}$$

where $cov_t = E_r\left[(\gamma_r - \bar{\gamma})(p_{r,t} - \bar{p}_t)\right]$.

We regress $\tilde{y}_{r,t} = \hat{\gamma}\tilde{p}_{r,t} + \nu_{r,t}$. The least squares moment condition is

$$E\left[\tilde{p}_{r,t}\left(\tilde{y}_{r,t} - \hat{\gamma}\tilde{p}_{r,t}\right)\right] = 0.$$

Substituting in:

$$\begin{aligned}
E\left[\tilde{p}_{r,t}\left((\bar{\gamma} - \hat{\gamma})\,\tilde{p}_{r,t} + (\gamma_r - \bar{\gamma})\, p_{r,t} - cov_t + \tilde{\varepsilon}_{r,t}\right)\right] &= 0 \\
(\bar{\gamma} - \hat{\gamma})\, E\left[\tilde{p}_{r,t}^2\right] + E\left[(\gamma_r - \bar{\gamma})\, p_{r,t}\tilde{p}_{r,t} - cov_t\tilde{p}_{r,t}\right] &= 0.
\end{aligned}$$

Note that cov_t has no variation over r and $\tilde{p}_{r,t}$ has no time-series variation. So we end up with:

$$\hat{\gamma} = \bar{\gamma} + \frac{E\left[(\gamma_r - \bar{\gamma})\, p_{r,t}\tilde{p}_{r,t}\right]}{E\left[\tilde{p}_{r,t}^2\right]}.$$