Appendix to ‘Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate’

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This appendix presents the firms optimality conditions and the complete set of competitive equilibrium conditions.

1 Firm’s Optimality Conditions

Write the Lagrangian of the firm as:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \rho_{t,a} \left\{ P_{i,a,t} (c_{i,a,t} + g_{i,a,t}) + P^*_{i,a,t} (c^*_{i,a,t} + g^*_{i,a,t}) - W_t (c_{i,a,t} + g_{i,a,t} + c^*_{i,a,t} + g^*_{i,a,t}) \right\}
\]

\[ + P_{a,t} \lambda^c_{i,a,t} \left( \frac{P_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^c_{i,a,t} + \theta^c_{a} \tilde{s}^c_{i,a,t-1} - c_{i,a,t} \]

\[ + P_{a,t} \lambda^g_{i,a,t} \left( \frac{P_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^g_{i,a,t} + \theta^g_{a} \tilde{s}^g_{i,a,t-1} - g_{i,a,t} \]

\[ + P^*_{a,t} \lambda^{c*}_{i,a,t} \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^{c*}_{i,a,t} + \theta^{c*}_{a} \tilde{s}^{c*}_{i,a,t-1} - c^*_{i,a,t} \]

\[ + P^*_{a,t} \lambda^{g*}_{i,a,t} \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^{g*}_{i,a,t} + \theta^{g*}_{a} \tilde{s}^{g*}_{i,a,t-1} - g^*_{i,a,t} \]

\[ + P_{a,t} \lambda^c_{i,a,t} \left( \frac{P_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^c_{i,a,t} + \theta^c_{a} \tilde{s}^c_{i,a,t-1} - (1 - \rho) c_{i,a,t} \]

\[ + P_{a,t} \lambda^g_{i,a,t} \left( \frac{P_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^g_{i,a,t} + \theta^g_{a} \tilde{s}^g_{i,a,t-1} - (1 - \rho) g_{i,a,t} \]

\[ + P^*_{a,t} \lambda^{c*}_{i,a,t} \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^{c*}_{i,a,t} + \theta^{c*}_{a} \tilde{s}^{c*}_{i,a,t-1} - (1 - \rho) c^*_{i,a,t} \]

\[ + P^*_{a,t} \lambda^{g*}_{i,a,t} \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{\eta} x^{g*}_{i,a,t} + \theta^{g*}_{a} \tilde{s}^{g*}_{i,a,t-1} - (1 - \rho) g^*_{i,a,t} \} \]

The optimality conditions associated with the firm’s problem with respect to \( P_{i,a,t}, P^*_{i,a,t}, c_{i,a,t}, \)
A competitive equilibrium is a set of processes that emerges by setting $\Theta_{a,i,t}^g$, $c_{i,a,t}^g$, $g_{i,a,t}^g$, $s_{i,a,t}^g$, $s_{i,a,t}^{c*}$, $s_{i,a,t}^{g*}$, respectively, are

$$
c_{i,a,t} + g_{i,a,t} = \eta \frac{P_{a,t}}{P_{i,a,t}} \left[ \nu_{i,a,t}^c(c_{i,a,t} - \theta_{a,i,a,t}^c) + \nu_{i,a,t}^g(g_{i,a,t} - \theta_{a,i,a,t}^g) \right]
$$

where $\nu_{i,a,t}^c$ and $\nu_{i,a,t}^g$ are the net inflows of commodities $c$ and $g$, respectively. The equilibrium conditions are:

$$
-P_{a,t}\nu_{i,a,t}^c + P_{i,a,t} - W_t - (1 - \rho)P_{a,t}\lambda_{i,a,t}^c = 0
$$

$$
-P_{a,t}\nu_{i,a,t}^g + P_{i,a,t} - W_t - (1 - \rho)P_{a,t}\lambda_{i,a,t}^g = 0
$$

$$
-P_{a,t}\nu_{i,a,t}^{c*} + P_{i,a,t} - W_t - (1 - \rho)P_{a,t}\lambda_{i,a,t}^{c*} = 0
$$

$$
-P_{a,t}\nu_{i,a,t}^{g*} + P_{i,a,t} - W_t - (1 - \rho)P_{a,t}\lambda_{i,a,t}^{g*} = 0
$$

$$
\theta_{a}^{c}E_{t+1}P_{a,t+1}\nu_{i,a,t+1}^c + P_{a,t}\lambda_{i,a,t}^c - \rho E_{t+1}P_{a,t+1}\lambda_{i,a,t+1}^c = 0
$$

$$
\theta_{a}^{g}E_{t+1}P_{a,t+1}\nu_{i,a,t+1}^g + P_{a,t}\lambda_{i,a,t}^g - \rho E_{t+1}P_{a,t+1}\lambda_{i,a,t+1}^g = 0
$$

$$
\theta_{a}^{c}E_{t+1}P_{a,t+1}\nu_{i,a,t+1}^{c*} + P_{a,t}\lambda_{i,a,t}^{c*} - \rho E_{t+1}P_{a,t+1}\lambda_{i,a,t+1}^{c*} = 0
$$

$$
\theta_{a}^{g}E_{t+1}P_{a,t+1}\nu_{i,a,t+1}^{g*} + P_{a,t}\lambda_{i,a,t}^{g*} - \rho E_{t+1}P_{a,t+1}\lambda_{i,a,t+1}^{g*} = 0
$$

Similar optimality conditions can be derived for foreign firms producing varieties of good $b$.

## 2 Competitive Equilibrium

A competitive equilibrium is a set of processes $x_{a,t}^c$, $x_{a,t}^g$, $x_{a,t}^{c*}$, $x_{a,t}^{g*}$, $x_{b,t}^c$, $x_{b,t}^g$, $x_{b,t}^{c*}$, $x_{b,t}^{g*}$, $s_{a,t}^c$, $s_{a,t}^g$, $s_{a,t}^{c*}$, $s_{a,t}^{g*}$, $s_{b,t}^c$, $s_{b,t}^g$, $s_{b,t}^{c*}$, $s_{b,t}^{g*}$, $\lambda_{a,t}^c$, $\lambda_{a,t}^g$, $\lambda_{a,t}^{c*}$, $\lambda_{a,t}^{g*}$, $\lambda_{b,t}^c$, $\lambda_{b,t}^g$, $\lambda_{b,t}^{c*}$, $\lambda_{b,t}^{g*}$, $\nu_{a,t}^c$, $\nu_{a,t}^g$, $\nu_{a,t}^{c*}$, $\nu_{a,t}^{g*}$, $\nu_{b,t}^c$, $\nu_{b,t}^g$, $\nu_{b,t}^{c*}$, $\nu_{b,t}^{g*}$, $\mu_{a,t}$, $\mu_{a,t}^{c*}$, $\mu_{b,t}$, $\mu_{b,t}^{c*}$, $\epsilon_{a,t}$, $\epsilon_{b,t}$, $\tau_t$, $g_t$, $e_t$, $y_t$, $tby_t$, and $c_t$ satisfying the complete asset market condition, the conditions defining the domestic block, and the conditions defining the foreign block, given initial conditions $s_{a,-1}^c$, $s_{a,-1}^g$, $s_{a,-1}^{c*}$, $s_{a,-1}^{g*}$, $s_{b,-1}^c$, $s_{b,-1}^g$, $s_{b,-1}^{c*}$, $s_{b,-1}^{g*}$, $g_{-i}$, $e_{-i}$, $y_{-i}$, $tby_{-i}$, and $c_{-i}$ for $i = 1, 2, 3, 4$, and the exogenous processes $\epsilon_t^1$, and $g_t^*$.  

### 2.1 Complete Asset Market Condition

$$
e_{a,t} = \frac{U_{x^c}(x_{a,t}^c, h_{t}^c)\chi_{a}(x_{a,t}^c, x_{b,t}^c)}{U_{x}(x_{a,t}^c, h_{t})\chi_{a}(x_{a,t}^c, x_{b,t}^c)} .
$$

### 2.2 Domestic Block

We introduce the parameter $\Theta^j_k$ for $j = c, g$ and $k = a, b$ to allow for the implementation of the superficial-habit model as a special case of the deep-habit model. Specifically, the deep-habit model emerges by setting $\Theta^j_k = \theta^j_k$, and the superficial-habit model obtains by setting $\theta^j_k = 0$. 


\[
\frac{\chi_b(x_{a,t}^c, x_{b,t}^c)}{\chi_a(x_{a,t}^c, x_{b,t}^c)} = \tau_t \\
x_t^c = \chi(x_{a,t}^c, x_{b,t}^c) \\
\frac{U_h(x_t^c, h_t)}{U_x(x_t^c, h_t) \chi_a(x_{a,t}, x_{b,t})} = w_t \\
c_{a,t} + c_{a,t}^* + g_{a,t} + g_{a,t}^* = h_t \\
x_{a,t}^c = g_{a,t} - \Theta_a^c s_{a,t-1}^g \\
x_{a,t}^c = c_{a,t} - \Theta_a^c s_{a,t-1}^c \\
x_{b,t}^g = b_{b,t} - \Theta_b^g s_{b,t-1}^g \\
x_{b,t}^c = c_{b,t} - \Theta_b^c s_{b,t-1}^c \\
1 - \frac{1}{\mu_a,t} = \nu_{a,t}^c + (1 - \rho) \lambda_{a,t}^c \\
1 - \frac{1}{\mu_a,t} = \nu_{a,t}^g + (1 - \rho) \lambda_{a,t}^g \\
1 - \frac{1}{\mu_a,t} = \nu_{a,t}^{c*} + (1 - \rho) \lambda_{a,t}^{c*} \\
1 - \frac{1}{\mu_a,t} = \nu_{a,t}^{g*} + (1 - \rho) \lambda_{a,t}^{g*} \\
c_{a,t} + g_{a,t} = \eta [\nu_{a,t}^c (c_{a,t} - \theta_a^c s_{a,t-1}^c) + \nu_{a,t}^g (g_{a,t} - \theta_a^g s_{a,t-1}^g)] \\
c_{a,t}^* + g_{a,t}^* = \eta [\nu_{a,t}^{c*} (c_{a,t}^* - \theta_a^{c*} s_{a,t-1}^{c*}) + \nu_{a,t}^{g*} (g_{a,t}^* - \theta_a^{g*} s_{a,t-1}^{g*})] \\
\theta_a^c \beta E_t \frac{U_x(t + 1) \chi_a(t + 1)}{U_x(t) \chi_a(t)} \nu_{a,t+1}^c + \lambda_{a,t}^c = \rho \beta E_t \frac{U_x(t + 1) \chi_a(t + 1)}{U_x(t) \chi_a(t)} \lambda_{a,t+1}^c \\
\theta_a^g \beta E_t \frac{U_x(t + 1) \chi_a(t + 1)}{U_x(t) \chi_a(t)} \nu_{a,t+1}^g + \lambda_{a,t}^g = \rho \beta E_t \frac{U_x(t + 1) \chi_a(t + 1)}{U_x(t) \chi_a(t)} \lambda_{a,t+1}^g \\
\theta_a^{c*} \beta E_t \frac{U_x^{c*}(t + 1) \chi_a^{c*}(t + 1)}{U_x^{c*}(t) \chi_a^{c*}(t)} \nu_{a,t+1}^{c*} + \lambda_{a,t}^{c*} = \rho \beta E_t \frac{U_x^{c*}(t + 1) \chi_a^{c*}(t + 1)}{U_x^{c*}(t) \chi_a^{c*}(t)} \lambda_{a,t+1}^{c*} \\
\theta_a^{g*} \beta E_t \frac{U_x^{g*}(t + 1) \chi_a^{g*}(t + 1)}{U_x^{g*}(t) \chi_a^{g*}(t)} \nu_{a,t+1}^{g*} + \lambda_{a,t}^{g*} = \rho \beta E_t \frac{U_x^{g*}(t + 1) \chi_a^{g*}(t + 1)}{U_x^{g*}(t) \chi_a^{g*}(t)} \lambda_{a,t+1}^{g*} \\
s_{a,t}^c = \rho s_{a,t-1}^c + (1 - \rho) c_{a,t} \\
s_{a,t}^g = \rho s_{a,t-1}^g + (1 - \rho) g_{a,t} \\
s_{a,t}^{c*} = \rho s_{a,t-1}^{c*} + (1 - \rho) c_{a,t}^{c*} \\
s_{a,t}^{g*} = \rho s_{a,t-1}^{g*} + (1 - \rho) g_{a,t}^{g*}
\[\mu_{a,t} = \frac{1}{w_t}\]  
(24)

\[\frac{\mu^*_a}{\mu_{a,t}} = e_{a,t}\]  
(25)

\[
\frac{c_{a,t} + g_{a,t} + e_{a,t}(c^*_{a,t} + g^*_{a,t})}{c_{a,t} + g_{a,t} + e_{a,t} + g^*_{a,t}}\ g_t = g_{a,t} + \tau t g_{b,t}
\]  
(26)

\[\frac{x^c_{a,t}}{x^c_{b,t}} = \frac{x^g_{a,t}}{x^g_{b,t}}\]  
(27)

\[
\ln(g_t/g) = B^1(L) \begin{bmatrix}
\ln(g_{t-1}/g) \\
\ln(y_{t-1}/y) \\
\ln(c_{t-1}/c) \\
\ln(a_{t-1}/a) \\
\ln(e_{t-1}/e)
\end{bmatrix} + \epsilon^1_t,
\]  
(28)

where \(w_t \equiv W_t/P_{a,t}\) denotes the real wage in terms of domestic goods.

### 2.3 Foreign Block

\[
\frac{\chi_b(x^c_{a,t}, x^c_{b,t})}{\chi_a(x^c_{a,t}, x^c_{b,t})} = \frac{\tau t e_{b,t}}{e_{a,t}}
\]  
(33)

\[x^c_t = \chi(x^c_{a,t}, x^c_{b,t})\]  
(34)

\[-\frac{U_h(x^c_t, h^*_t)}{U_a(x^c_{a,t}, x^c_{b,t})} = w^*_t\]  
(35)

\[c_{b,t} + g_{b,t} + e_{b,t} + g^*_b = z^*_t h^*_t\]  
(36)

\[x^c_{a,t} = c^*_{a,t} - \Theta^c a s_{a,t-1}\]  
(37)

\[x^g_{a,t} = g^*_{a,t} - \Theta^g a s^g_{a,t-1}\]  
(38)

\[x^c_{b,t} = c^*_b - \Theta^c b s^c_{b,t-1}\]  
(39)

\[x^g_{b,t} = g^*_b - \Theta^g b s^g_{b,t-1}\]  
(40)

\[\frac{\mu_{b,t} - 1}{\mu_{b,t}} = \nu^c_{b,t} + (1 - \rho)\lambda^c_{b,t}\]  
(41)
\[
\frac{\mu_{b,t} - 1}{\mu_{b,t}} = \nu_{b,t}^g + (1 - \rho)\lambda_{b,t}^g \quad (42)
\]
\[
\frac{\mu_{b,t}^* - 1}{\mu_{b,t}^*} = \nu_{b,t}^{c*} + (1 - \rho)\lambda_{b,t}^{c*} \quad (43)
\]
\[
\frac{\mu_{b,t}^{g*} - 1}{\mu_{b,t}} = \nu_{b,t}^{g*} + (1 - \rho)\lambda_{b,t}^{g*} \quad (44)
\]
\[
c_{b,t} + \nu_{b,t} = \eta[\nu_{b,t}^c(c_{b,t} - \theta_b^c s_{b,t-1}) + \nu_{b,t}^g(g_{b,t} - \theta_b^g s_{b,t-1})] \quad (45)
\]
\[
c_{b,t}^* + \nu_{b,t} = \eta[\nu_{b,t}^{c*}(c_{b,t}^* - \theta_b^{c*} s_{b,t-1}) + \nu_{b,t}^{g*}(g_{b,t} - \theta_b^{g*} s_{b,t-1})] \quad (46)
\]
\[
\theta_b^c \beta E_t \frac{U_x(t + 1)\chi_b(t + 1)}{U_x(t)\chi_b(t)} \nu_{b,t+1}^c + \lambda_{b,t}^c = \rho \beta E_t \frac{U_x(t + 1)\chi_b(t + 1)}{U_x(t)\chi_b(t)} \lambda_{b,t+1}^c \quad (47)
\]
\[
\theta_b^g \beta E_t \frac{U_x(t + 1)\chi_b(t + 1)}{U_x(t)\chi_b(t)} \nu_{b,t+1}^g + \lambda_{b,t}^g = \rho \beta E_t \frac{U_x(t + 1)\chi_b(t + 1)}{U_x(t)\chi_b(t)} \lambda_{b,t+1}^g \quad (48)
\]
\[
\theta_b^{c*} \beta E_t \frac{U_x^*(t + 1)\chi_b^*(t + 1)}{U_x^*(t)\chi_b^*(t)} \nu_{b,t+1}^{c*} + \lambda_{b,t}^{c*} = \rho \beta E_t \frac{U_x^*(t + 1)\chi_b^*(t + 1)}{U_x^*(t)\chi_b^*(t)} \lambda_{b,t+1}^{c*} \quad (49)
\]
\[
\theta_b^{g*} \beta E_t \frac{U_x^*(t + 1)\chi_b^*(t + 1)}{U_x^*(t)\chi_b^*(t)} \nu_{b,t+1}^{g*} + \lambda_{b,t}^{g*} = \rho \beta E_t \frac{U_x^*(t + 1)\chi_b^*(t + 1)}{U_x^*(t)\chi_b^*(t)} \lambda_{b,t+1}^{g*} \quad (50)
\]
\[
s_{b,t}^c = \rho s_{b,t-1}^c + (1 - \rho)c_{b,t} \quad (51)
\]
\[
s_{b,t}^g = \rho s_{b,t-1}^g + (1 - \rho)g_{b,t} \quad (52)
\]
\[
s_{b,t}^{c*} = \rho s_{b,t-1}^{c*} + (1 - \rho)c_{b,t} \quad (53)
\]
\[
s_{b,t}^{g*} = \rho s_{b,t-1}^{g*} + (1 - \rho)g_{b,t} \quad (54)
\]
\[
\frac{\mu_{b,t}^*}{\mu_{b,t}} = \frac{s_{b,t}^*}{w_t^*} \quad (55)
\]
\[
\frac{\mu_{b,t}^{g*}}{\mu_{b,t}} = c_{b,t} \quad (56)
\]
\[
g_{a,t} + \tau_{e_{a,t}} \frac{e_{a,t}^g g_{b,t}^*}{e_{a,t}^g + g_{b,t}^*} = \frac{\tau_{e_{a,t}}(c_{b,t} + g_{b,t}) + e_{b,t} \tau_{e_{a,t}} c_{b,t}^* + g_{b,t}^*}{c_{b,t} + g_{b,t} + c_{b,t}^* + g_{b,t}^*} \quad (57)
\]
\[
\frac{x_{a,t}^{c*}}{x_{b,t}^{c*}} = \frac{x_{a,t}^{g*}}{x_{b,t}^{g*}} \quad (58)
\]

where \( w_t^* \equiv W_t^* / P_{b,t}^* \).