Internet Appendix for Taxes on Tax-Exempt Bonds

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This Internet Appendix for the paper "Taxes on Tax-Exempt Bonds" makes available several additional results. Section 1 discusses the taxation of municipal bonds sold prior to maturity. We discuss in particular the ex-ante incentives to defer the sale of a premium and par bond to maturity and show this incentive is small, at less than one basis point. Section 2 discusses further details of constructing the municipal zero curve described in Appendix C of the paper. In Section 3, we report the results of additional analysis and robustness checks.

1 Market Discount Taxation of Bonds Sold Prior to Maturity

The general principle involved in the taxation of market discount for bonds not held to maturity is that there is a second accretion schedule that applies to the purchase price of the bond, which is computed using the yield at the time of purchase. Any resulting capital gain or loss is computed relative to this second accrual of discount, sometimes termed total discount. For an investor buying a par or a premium bond, there is no ex-ante incentive to sell the bond early assuming that the expected return on the bond is equal to its purchase yield (or a version of the Expectations Hypothesis [EH] holds for purchasing that municipal bond). While the EH is rejected for Treasury yields, the R^2 s of predictability regressions are typically very low (see Campbell and Shiller ((1991), and it is not clear individual investors are able to successfully market time in the municipal bond market to take advantage of these deviations from the EH. Certainly, retail investors trying to time the market generally underperform aggregate market benchmarks in equity market trades (see, for example, Barber and Odean (2000)). Early evidence by Brick and Thompson (1978) finds lagged Treasury yields do not predict long-term municipal yields. We first consider the case of original issue premium and par bonds in Section 1.1 and then discuss OID bonds in 1.2.

1.1 Premium and Par Bonds

1.1.1 Tax Treatment

Consider a bond with an original 10-year maturity with a 10% coupon. We refer to this bond as Bond A. Suppose two years after issue at t=2, Bond A trades at a price of \$95. The market discount on this bond at the time of purchase is 100-95=\$5. Suppose that at t=8, the bond is sold for \$99. Thus, from t=2 to t=8, the investor has made a gain of 99-95=\$4. A portion of this \$4 gain is taxed as ordinary income and a portion is taxed as a capital gain.

When Bond A is purchased at t=2 for \$95, the investor purchasing Bond A does so at a yield of 10.9543%. As Bond A approaches maturity, the price of Bond A would deterministically approach 100 under the constant yield method if the yield was constant at 10.9543%. This accretion of discount is taxed as income under IRC § 1276. Anything in excess of this accretion is defined as a capital gain. At t=8, the revised price of Bond A, computed as the present value of the remaining four 5% coupons at a yield of 10.9543%/2 = 5.4772% every six months is \$98.3266. The accretion of market discount of 98.3266 - 95 = \$3.3266 is counted as income. Thus, 4-3.3266 = \$0.6734 is taxed as income.

We illustrate this in the top panel of Figure 1. The accretion of market discount at the purchase yield of 10.9543% is shown in the dashed red line. This accretion is taxed as income. At t=8, the difference between the sale price of \$99, denoted as the red square, and the accreted purchase price of Bond A of \$98.3266 is \$0.6734 and it is taxed as a capital gain.

The accretion of market discount affects the tax basis of the bond. Any sale of Bond A below the accreted market discount can be deducted as a loss. This also applies to purchases at a premium. While the amortization of municipal bond premiums as a result of secondary market transactions cannot be deducted, the amortization of the premium changes the tax basis of the bond.

The top panel of Figure 1 also shows the de minimis boundary. Any purchase above the de minimis threshold results in no market discount. At t=2, Bond A's de minimis boundary is $100(1-0.0025\times8)=\$98$. Suppose that at t=2, Bond A is purchased for \$98.50 and sold at t=8 for \$99. Then, the 99-98.50=\$0.50 profit is taxed entirely as a capital gain.

1.1.2 Ex-Ante Incentive to Sell Later

If the yield on the purchased bond remains constant at y, then a par or premium bond purchased in the secondary market would never be sold early. In this environment, a par or premium bond's value would rise in accordance with the revised price schedule, which changes according to

$$P_n = P_{n-1}(1 + y/2) - 100 \times C/2,$$

in each subsequent six-month period, with P_0 being the clean purchase price. For a par or premium bond, y > C and the revised purchase price accelerates as the bond approaches maturity. Taxes are not paid each period on $P_n - P_{n-1}$, which would be the timing appropriate if taxes are levied when the economic income is earned. Instead, taxes are levied on $P_n - P_0$ only at the time of sale. Not surprisingly, it is straightforward to show that pushing out the payment

of the tax increases the after-tax value of the cashflows valued at y, and thus the after-tax yield increases as the sale is deferred.

The value to waiting is economically very small. For example, suppose that the income tax rate is 35%. If a par bond with an initial coupon of C=1% and a remaining 30-year maturity is purchased at a tax-exempt yield of 20%, then the after-tax yield if the bond is sold after 6 months is 19.9771%, compared to 19.9773% if the bond is held to maturity, assuming in both cases that the yield curve remains flat at y. Higher tax rates, shorter maturities, or lower coupon rates (initial yields) will cause these differences to be even smaller.

There is one counter-argument for an investor to sell a par or premium bond early. In upward-sloping yield curve environments, which has always been the case in the municipal market, Bankman and Klein (1989) show that the constant yield accrual method understates interest income in the early years and there is too much accrual in later years. If yields are held constant, selling early would result in a lower tax payment. However, Sims (1992) numerically shows that such effects are very small, but they would off-set the incentive for investors to not sell early to defer tax.

1.2 OID Bonds

1.2.1 Tax Treatment

The taxation of market discount for OID bonds depends on the OID accrual schedule relative to the accrual schedule for the market discount. This is best illustrated through an example. Consider Bond B, which is an OID bond originally issued with a 10-year maturity paying a 10% coupon. Bond B was issued at a price of \$88.5301 with a par value of 100 and initial yield of 12%. Suppose that at time t=2, Bond B is purchased for \$84, corresponding to a yield of 13.3105%, and at t=8, Bond B is sold for \$99. The computation of taxable market discount is complicated by the fact that the accreted OID is not taxable.

The bottom plot of Figure 1 illustrates this case. The solid blue line plots the accretion of OID at the original issue yield of 12%. These are the revised prices of Bond B according to the OID accrual schedule. At t=2, the purchase price is denoted by a red diamond, and this purchase price is accreted at the purchase yield of 13.3105% in the dashed red line. In Figure 1, the vertical line at t=2 is the difference between the purchase price of \$84 and the revised price of \$89.8941.

At t = 8, Bond B is sold for \$99, which we denote on Figure 1 by the red square. Normally,

the accretion of the purchase price from t=2 to t=8 along the dashed red curve would be subject to income tax. However, some of this accretion is exempt from tax because accretion would have happened under the OID accrual schedule. It is only the accretion of the market discount in excess of the OID accretion that is taxable.

From t=2 to t=8, the revised price of Bond B under the OID schedule increases from \$89.8941 to \$96.5349. The OID accrual schedule can be computed by valuing the remaining payments of Bond B at the original issue yield of 12%. This difference in OID of 89.8941-96.5349=\$6.6408 is not taxable. When Bond B is purchased at t=2, we apply the constant accrual yield method to compute the accrual of total discount at the purchase yield of 13.3105%. At t=8, we can compute this accreted discount by valuing the remaining four payments of \$5 every six months plus the final par value of \$100 at a yield of 6.6553% every six months, which totals \$94.3495. According to the total discount schedule, there is an increase of 94.3495-84=\$10.3495. However, according to the OID schedule, we would originally have seen an increase in the bond price of \$6.6408. The taxable market discount is the difference in the accrued OID and the total discount accrual, so only 10.3495-6.6408=\$3.7087 is subject to income tax.

In Figure 1, the difference between the two vertical lines represents the accrued market discount at the time of sale. The vertical line at t=2 is the market discount at purchase, which is 89.8941-84=\$5.8941. The vertical line at t=8 is the market discount at sale, which is 96.5349-94.3495=\$2.1854. The difference between these two vertical lines is the accrued market discount, which is 5.8941-2.1854=\$3.7087. According to IRC § 1276(a)(1), only this portion of the total gain is treated as ordinary income.

To summarize, the total gain involved in purchasing Bond B at t=2 for \$84 and selling Bond B at t=8 for \$99 is \$15. The OID accrual from t=2 to t=8 of \$6.6408 is not taxable. Thus, the total taxable income is 99-84-6.6408=\$8.3592 at the time of sale. At t=8, the investor owes income tax on \$3.7087 and the remainder of \$4.6505 is taxed as a capital gain.

For a second case, suppose that at t=2, Bond B is trading for \$89, which is above the de minimis boundary of Bond B at t=2, which is $DM=89.8941-100\times0.0025\times8=\87.8941 . Bond B is sold at t=8 for \$99. Bond B is not subject to market discount as the purchase price is above the de minimis threshold, but below the revised price of \$89.8941 at t=2. Thus, no income tax is payable but some of the 99-89=\$10 gain is taxable as a capital gain. According to the OID schedule, \$6.6408 is not taxable, so the total taxable income is 99-89-6.6408=\$3.3592, which is taxed at the capital gains rate.

As a final case, suppose that at t=2, Bond B trades above its accreted OID schedule for

\$91. Again suppose that Bond B is sold at t=8 for \$99. In this case, no income tax on the gain is paid and since some of the OID accretion is tax-free, not all of the 99-91=\$8 is taxed. From t=2 to t=8, the change in the OID schedule is \$6.6408, so the total taxable income is 99-91-6.6408=\$1.3592 on which capital gains tax is owed.

1.2.2 Ex-Ante Incentive to Sell Early

With OID bonds, there is a difference between the OID accrual and the purchase price accrual. Because the purchase yield is larger than the initial yield, the accrual of the total discount on the purchase price is faster than the accrual of OID. Since OID is tax-exempt, there is an incentive to bring forward the sale of the bond. This does not occur for a par or premium bond because the revised price is always fixed at par value. However, even if investors expect to receive the purchase yield over the remaining maturity of the bond, there is an incentive for investors to sell an OID bond early because of the different accretion schedules of OID and total discount.

For realistic cases, the additional advantage of selling early is worth less than 1 basis point. Consider an OID bond with original coupon rate 2.35% with an original issue yield of 5% and a remaining maturity of 10 years. We assume the tax-exempt yield curve is flat at 6% and remains constant. If the income tax rate is 0.396, then the after-tax yield assuming a sale in six months is 5.6971% compared to 5.6954% if the investor holds the bond to the full 10-year maturity.

2 Municipal Zero-Coupon Bond Curves

Figure 2 plots the time series of our estimated municipal zero-coupon curves for maturities of one, five, and 10 years in the top two panels and the bottom left panel. Along with our estimates, we also plot the zero-coupon curve for municipal bonds from Bloomberg, which start in August 2001. Bloomberg's method of computing zero rate curves is not made public, but Bloomberg's sample includes bonds with callable and sinking fund features. Bloomberg's estimation sample also relies on dealer quotations in addition to using transaction prices. Our zero-coupon rates are slightly lower than the Bloomberg data, which is expected because our sample excludes all bonds with embedded option features. Yields at the 1-year maturity hovered around 1% from 1995 to 1999, and rose to 5% in 2000 before declining to 1% in 2003. At the end of our sample in November 2005, the 1-year zero municipal yield was around 3%. At the long end of the yield curve, 10-year zero yields have been much more stable, generally around 4-5% over 1995 to 2007. These trends mirror the movements in Treasury yields.

In the lower right panel of Figure 2, we plot the average term structure of municipal zero-coupon rates over our sample from one to 10 years. It is a well-known stylized fact that municipal yield curves have always been upward sloping (see Fabozzi and Feldstein (1983)). We find our estimated zero yield curves are upward sloping in every trading day. The average term spread between one- and 10-year zero-coupon yields is 1.27%. To compare this number to par coupon yields, we also plot the average par coupon yield curve implied from our zero yields. The corresponding par yield spread is 1.18%.

3 Additional Analysis

In this section, we report results of additional tests and controls. Section 3.1 considers matching below de minimis trades with fully tax-exempt trades controlling for transaction size. Section 3.2 investigates the empirical duration of market discount bonds. In Section 3.3, we consider other potential characteristics which may differentiate the effects of retail versus institutional trades.

3.1 Matching by Transaction Size

Table 1 considers matching by transaction size as an additional liquidity control. For each trade with prices greater than \$1.00 below de minimis (bin 2), we match trades in bin 2 with a trade of exactly the same par amount traded among bonds trading at least \$1.00 above their revised prices (bin 1). If no matching trade is found in bin 1, we do not consider that transaction. If there is more than one trade in bin 1 with the same par value traded as the trade in bin 2, we take a random trade of the same transaction size. Table 1 shows that controlling for transaction produces a difference in after-tax yield spreads of 26 basis.

3.2 Empirical Duration

Besides yield or price levels, municipal bond traders often consider duration risk in the management of their portfolios. The higher yields on below de minimis bonds may represent compensation for bearing bonds with duration that is not captured by the zero curve. Managing these duration exposures may be important for tracking, or benchmarking, against index positions.

In Table 2, we compare empirical duration with model-implied durations. We compute the empirical duration for a bond if the previous trade of the bond occurs within the past 30 days

and the price level of the last trade occurs in the same tax region as the current trade. Bins 1-3 represent the no-tax region, bin 4 is the capital gains tax region, and bins 5-7 are the income tax regions. The empirical duration is computed as

$$D_t = \frac{(P_t - P_{last})/P_{last}}{Y_t - Y_{last}},\tag{1}$$

where P_t is the current trade price of the bond with tax-exempt yield Y_t and P_{last} is the price of the bond within the last 30 days with tax-exempt yield Y_{last} . Similarly, we compute a model-implied duration:

$$D_t^m = \frac{(P_t^m - P_{last}^m)/P_{last}^m}{Y_t^m - Y_{last}^m},$$
(2)

which instead uses the bond prices and tax-exempt yields implied by the zero coupon yield curve. We report the empirical and model duration for each of the seven price bins. We are particularly interested in the duration spread $D - D^m$, which is the empirical duration not captured by the theoretical zero curve.

Table 2 reports that duration in excess of the zero curve is larger for bonds well in the notax region than for deep below de minimis bonds. For bin 1 with prices greater than \$1.00 above revised price, $D - D^m$ is 2.12, whereas bonds in bin 7 trading at least \$1.00 below de minimis have a duration spread of 1.62. These patterns are also observed when only interdealer trades are considered, where bonds in bins 1 and 7 have almost the same duration spreads, with $D - D^m$ being 1.86 and 1.70, respectively. Thus, the higher yields for below de minimis bonds is not due to these bonds having higher empirical duration than the zero curve implies compared to municipal bonds without market discount.

3.3 Original Issue Price

A bond characteristic that potentially separates retail investors from institutions is the original issue price. Anecdotal evidence from several municipal bond portfolio managers and dealers suggests retail investors are drawn more to original premium and par issues over OID bonds. While these investor preferences are for bonds at issue, we find little effect of the original issue price on the de minimis premium.

Table 3 reports after-tax yield spreads for retail and institutional transactions subdivided by original issue price. Consistent with Table 5 of the paper, institutional trades do not exhibit a de minimis premium. Institutional trades of market discount bonds which are original issue par bonds have yields 6 basis points lower than fully tax-exempt equivalent bonds. Table 3 shows

there is little effect of the original issue price. For retail trades, the after-tax yield difference between bonds trading above revised price and bonds trading below de minimis is 21 basis points for original issue premium bonds, 16 basis points for original issue par bonds, and 22 basis points for OID bonds. In summary, even though retail clients may be initially attracted to premium and par bonds at issue, when bond prices drop below the de minimis boundary, all bonds are assigned a de minimis premium regardless of their original terms of issue.

References

- [1] Bankman, J., and W. A. Klein, 1989, "Accurate Taxation of Long-Term Debt: Taking into Account the Term Structure of Interest," *NYU Tax Law Review*, 44, 335-348.
- [2] Barber, B., and T. Odean, 2000, "Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors," *Journal of Finance*, 55, 773-806.
- [3] Brick, J. R., and H. E. Thompson, 1978, "Time Series Analysis of Interest Rates: Some Additional Evidence," *Journal of Finance*, 33, 93-103.
- [4] Campbell, J. Y., and R. J. Shiller, 1991, "Yield Spreads and Interest Rate Movements: A BirdŠs Eye View, Review of Economic Studies," 58, 495-514.
- [5] Fabozzi, F., and S. Feldstein, 1983, "Municipal Bonds," in F. Fabozzi and T. M. Pollack, eds., *Handbook of Fixed Income Securities*, Dow Jones-Irwin, Homewood, IL.
- [6] Fama, Eugene F., and James D. MacBeth, 1973, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 71, 607-636.
- [7] Sims, T. S., 1992, "Long-Term Debt, the Term Structure of Interest and the Case for Accrual Taxation," *NYU Tax Law Review*, 47, 313-375.

Table 1: Controlling for Transaction Size

	Bin 1	2	
	>1	≤-1	# of Days
All Trades	1.59	27.83	2095
	(0.20)	(1.39)	
# Trades per Day	105	105	
Average Par Amount Traded per Day	\$63,465	\$63,465	

The table reports average after-tax yield spreads (the difference between the after-tax transaction yield and the model-implied yield) for bonds of different types partitioned into different price bins, representing default risk controls controlling for transaction size in terms of par amount traded. Each day, we match trades of bonds with prices greater than \$1.00 above revised price (bin 1) with trades of the same par amount traded of bonds with prices lower than \$1.00 below the de minimis boundary (bin 2). If no matching trade is found, we do not consider that transaction. If there is more than one trade in the latter category with the same par value traded as the trade in the former category, we take a random trade of the same transaction size. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the number of days for each type of trade used to compute the average and the average number of series per day in each price bin. We report standard errors in parentheses computed following the procedure of Fama and MacBeth (1973). The sample period is from January 1995 to December 2005.

Table 2: Empirical Duration Across Different Price Bins

	Bin 1 >1	2 (0.5 1]	3 [0 0.5]	$4 \\ (RP DM)$	5 (-0.5 0]	6 (-1 -0.5]	7 <-1
All Trades						·	
Empirical Duration D Model Duration D^m $D-D^m$	7.00 4.88 2.12 (0.09)	4.58 4.17 0.41 (0.16)	3.47 4.15 -0.68 (0.04)	6.31 5.50 0.81 (0.13)	6.62 4.16 2.46 (0.51)	6.52 4.59 1.93 (0.50)	7.31 5.70 1.62 (0.19)
Interdealer Trades							
Empirical Duration D Model Duration D^m $D-D^m$	6.99 5.13 1.86 (0.13)	4.62 4.30 0.31 (0.11)	3.87 4.24 -0.37 (0.07)	5.57 5.43 0.14 (0.12)	6.39 4.20 2.19 (0.64)	5.78 4.60 1.18 (0.27)	7.34 5.64 1.70 (0.30)

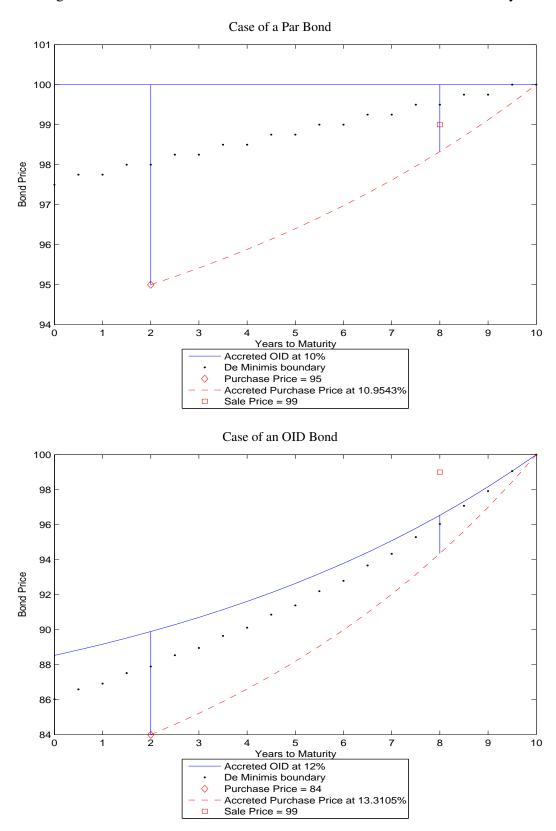
In the table we report the empirical duration and model duration of bonds partitioned into different price bins. The seven bins are based on the distance between the bond price and the revised price (RP) or de minimis boundaries (DM). The seven bins are based on the distance between the bond price and the revised price (RP) or de minimis boundaries (DM). The seven bins containing bond trades with prices higher than RPare defined as: bin 1 (>1) with prices greater than 1.00 above RP; bin 2 (0.5,1] with prices between 0.50and \$1.00 (including \$1) dollar above RP; bin 3 [0 0.5] with prices between \$0 and \$0.50 (including \$0 and \$0.50) above RP; bin 4 (RP DM) which includes trades with prices between RP and DM; bin 5 (-0.5 0] with prices from \$0.00 to \$0.5 (including \$0.00) below DM; bin 6 (-1 -0.5] with prices from \$0.50 to \$1.00 below DM; and the last bin 7 (< -1) contains all the prices more than or equal to \$1.00 below DM. Bins 1-3 represent the no-tax region, bin 4 is the capital gains tax region, and bins 5-7 are the income tax regions. The empirical duration of a trade is computed if the previous trade of the bond occurs within the past 30 days and the price level of the last trade occurs in the same tax region as the current trade following equation (1). Similarly, the model-implied duration is computed using bond prices and yields implied from the zero coupon yield curve using equation (2). We report standard errors of the difference between empirical duration and model implied duration, $D-D^m$, in parentheses. The sample period is from January 1995 to December 2005.

Table 3: Transactions Sorted by Original Issue Price

		Retail Trades			Institutional Trades		
		$Y_{ au}-Y^m$	SE	# Trades per Day	$Y_{\tau} - Y^m$	SE	# Trades per Day
Premium	$(1) \ge RP$ $(2) \le DM$ (2)- (1)	-6.98 13.78 20.76	(0.09) (1.19)	936 74	-12.04 -12.28 -0.25	(0.13) (1.10)	434 17
Par	$(1) \ge RP$ $(2) \le DM$ (2)- (1)	1.03 16.84 15.81	(0.13) (1.04)	164 41	-9.84 -15.80 -5.96	(0.21) (0.68)	29 9
Discount	$(1) \ge RP$ $(2) \le DM$ (2)- (1)	-0.65 21.38 22.04	(0.09) (0.98)	481 93	-9.70 -7.57 2.13	(0.12) (0.91)	84 18

The table reports average after-tax yield spreads (the difference between the after-tax transaction yield, Y_{τ} , and the model-implied yield, Y^m) in basis points for bond transactions sorted by original issue price (premium, par, or discount). A transaction is defined as a retail trade if the par amount traded is below \$100,000 and defined as a institutional trade otherwise. In each case, we partition trades into two bins based on the revised price (RP) and de minimis boundaries (DM). Bin $1, \geq RP$, contains all transactions with prices above RP where there is no tax liability involved. Bin $2, \leq DM$, contains all the trades with prices below DM, which are subject to income tax. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the average number of trades per day for each bin. We report standard errors (SE) in parentheses computed using the method of Fama and MacBeth (1973). The sample period is from January 1995 to April 2007.

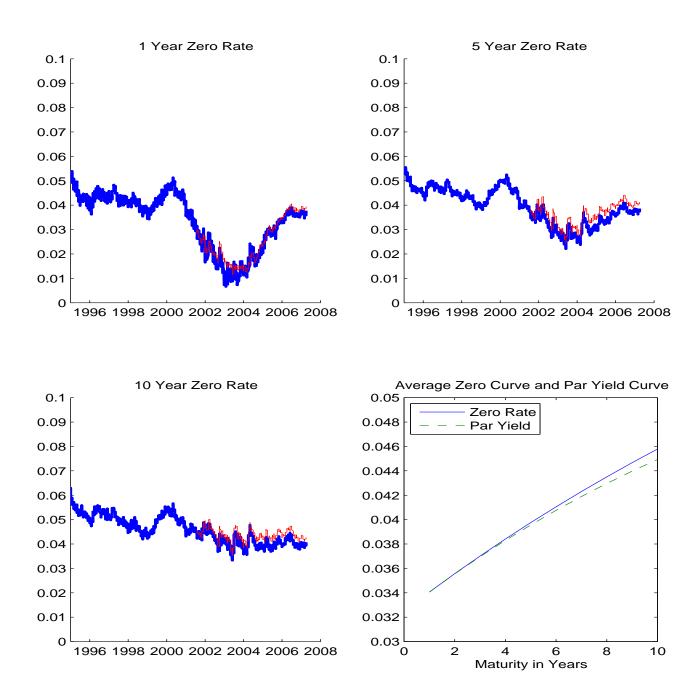
Figure 1: Illustration of Market Discount for Bonds Sold Prior to Maturity



Note to Figure 1

The figure illustrates the taxation of market discount for a bond sold prior to maturity. We consider the case of a par bond in the top panel and an OID bond in the bottom panel. In the top panel, consider a par bond originally issued with a 10-year maturity paying a 10% semi-annual coupon (Bond A). The top panel illustrates the case of a market transaction comprising both income and capital gain components. At t=2, Bond A is sold at a price of \$95 representing a yield of 10.9543%. This point is denoted as a diamond in the figure. The dashed line plots the accreted discount of the \$95 purchase price (the revised price) to maturity at a yield of 10.9543%. This accretion is taxed as income. At t=8, the bond is sold for a price of \$99, which is denoted by a square. The revised price of Bond A at t = 8 is \$98.3266. The gain in excess of the accretion of market discount is the distance between the red square and the dashed line, which is \$0.6734, which is taxed as a capital gain. The remaining \$3.3266, which is the accretion of market discount, is taxed as income. In the bottom panel, consider Bond B, which is an OID bond originally issued with a 10-year maturity paying a 10% coupon. Bond B was issued at a price of \$88.5301 with a par value of 100 with an initial yield of 12%. The accretion of OID from t=0 is shown in the solid blue line. Suppose that at time t=2, Bond B is purchased for \$84 at a yield of 13.3105%, which is denoted by the red diamond. The accretion of this purchase price at 13.3105% is shown in the dashed red line. At t = 8, Bond B is sold for \$99, denoted in the figure as a red square. The first vertical line at t=2 represents the market discount at purchase. The second vertical line at t=8 represents the market discount at sale. The difference between the two vertical lines represents the accrued market discount at the time of sale, which is taxed as income at the time of sale.

Figure 2: Municipal Zero-Coupon Yield Curves



The plot displays the estimated municipal zero-coupon yield curves for our A-grade municipal bond sample. The construction method is outlined in Appendix ??. In the top two plots, as well as the bottom left plot, we display the estimated zero curves in solid, heavy lines for various maturities. For comparison, we overlay Bloomberg's AAA yields in the red, light lines from August 2001. In the bottom right plot, we graph the average zero yield curve and the average par yield curve over the January 1995 to April 2007 sample.