

How does macroprudential regulation change bank credit supply?

Online Appendix *

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May 2014

Microfoundation of the function form of the bank-run probability

In the paper we have assumed that the probability of a bank-run, q , is a decreasing function of $\theta \equiv \frac{LIQ_1 + \xi \cdot I}{D^R(1 + r_2^D)}$. The purpose of the online appendix is to present the microfoundations that justify our assumption. We follow the of Goldstein and Pauzner (2005) who apply the global game techniques of Carlsson and van Damme (1993) and Morris and Shin (1998) in bank-run models. Section 1 derives q , while section 2 discusses existence and uniqueness. We show how our framework satisfies the assumptions underlying the existence and uniqueness of a threshold equilibrium, and refer the reader to the aforementioned papers for details.

1 Derivation of threshold equilibrium

Assume that the probability of the state of the world, which is realized at $t = 3$, is driven by a state variable z_τ , $\tau \in \{1, 2\}$. Define a function $\omega_{3s} : z_\tau \rightarrow [0, 1]$ such that $\frac{\partial \omega_{3g}(z_\tau)}{\partial z_\tau} > 0$, $\frac{\partial \omega_{3m}(z_\tau)}{\partial z_\tau} \geq 0$ and

*We are grateful to John Geanakoplos, Frank Smets and seminar participants at the Norges Bank, Board of Governors of the Federal Reserve, Wisconsin School of Business, Chicago Booth, the International Monetary Fund, the Bank of England and the 10th Annual GE Cowles conference for comments. Kashyap thanks the Initiative on Global Markets at the University of Chicago Booth School of Business and the National Science Foundation for a grant administered by the National Bureau of Economic Research for research support. Kashyap's disclosures of his outside compensated activities are available on his web page. All errors herein are ours. The views expressed in this paper are those of the authors and do not necessarily represent those of Federal Reserve Board of Governors, anyone in the Federal Reserve System, or any of the institutions with which we are affiliated.

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$\frac{\partial \omega_{3b}(z_\tau)}{\partial z_\tau} < 0$. All agents have common priors z_1 at $t = 1$. Also, assume that $z_2 = z_1 + \eta$ at $t = 2$, where $\eta \sim U[-\bar{\eta}, \bar{\eta}]$. Also, $\mathbb{E}(\omega_{3s}(z_2)|z_1) = \omega_{3s}(z_1)$. We assume that η is realized at the beginning of period 2, but it is not publicly revealed. Rather, each depositor obtains a signal $x_i = \eta + \varepsilon_i$, where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, \varepsilon]$. The signal provides information regarding the expected outcomes in period 3: The higher the signal, the higher is the posterior probability attributed by the agent to the event that the bank will not go bankrupt and that deposits will be paid in full, and the lower the incentive to run on the bank. In addition, an agent's signal provides information about other agents' signals, which forms the basis for an inference regarding their actions. Observing a high signal makes the agent believe that other agents obtained high signals as well. Consequently, if R receives a high signal, he attributes a low likelihood to the possibility of a bank-run. This makes the incentive to run even smaller.

Consider portfolio decisions to be predetermined. While all impatient depositors demand early withdrawal, patient ones need to compare the expected payoffs from going to the bank in period 2 or 3. The ex-post payoff of a patient agent from these two options depends on both η and the proportion m of agents demanding early withdrawal (defined below).

We are interested in a threshold equilibrium in which a patient depositor with signal x_i withdraws his deposits at $t = 2$ when the signal is below a common threshold, i.e., $x_i \leq x^*$. Otherwise, he withdraws at $t = 3$. This implies also a threshold for the fundamentals, i.e., a run will occur when $\eta \leq \eta^*$.

The analysis proceeds in two steps:

1. Calculate η^* given x^* .
2. Calculate x^* given η^* .

Denote by $m(\eta, x^*)$ the total proportion of withdrawals at $t = 2$ when fundamentals are η and the signal threshold is x^* .

Step 1

The liquidation value of the bank's assets at $t = 2$ is $LIQ_1 + \xi \cdot I$, while the total promised deposit repayment is $D^R(1 + r_2^D)$. The bank is liquidated at $t = 2$ if $m(\eta, x^*)D^R(1 + r_2^D) \geq LIQ_1 + \xi \cdot I$. Recall that $\theta = \frac{LIQ_1 + \xi \cdot I}{D^R(1 + r_2^D)}$ is the probability of being paid if all depositors run on the bank. Thus,

a bank-run occurs if

$$m(\eta, x^*) \geq \theta. \quad (1)$$

The next step is to derive the distribution of $m(\eta, x^*)$. If $\eta < x^* - \varepsilon$ all agents receive signals below the threshold x^* and everyone runs. If $\eta > x^* + \varepsilon$, all agents receive signals above the threshold, all patient depositors wait and only impatient depositors withdraw. In the intermediate range for fundamentals, the proportion of patient depositors who withdraw depends on the probability that they receive a signal below threshold, which is equal to $Prob(x_i \leq x^*) = \frac{x^* - \eta + \varepsilon}{2\varepsilon}$. Thus,

$$m(\eta, x^*) = \begin{cases} 1 & \text{if } \eta < x^* - \varepsilon \\ \delta + (1 - \delta) \frac{x^* - \eta + \varepsilon}{2\varepsilon} & \text{if } x^* - \varepsilon \leq \eta \leq x^* + \varepsilon \\ \delta & \text{if } \eta > x^* + \varepsilon \end{cases} \quad (2)$$

For $\eta = \eta^*$, $m(\eta^*, x^*) = \theta$ and

$$\eta^* = x^* + \varepsilon \left(1 - 2 \frac{\theta - \delta}{1 - \delta} \right). \quad (3)$$

The previous equation solves for η^* as a function x^* .

Step 2

x^* is the signal threshold at which a patient depositor is indifferent between withdrawing and waiting in period 2. We need to compute the utility differentials between waiting and running as a function of η and $m(\eta, x^*)$. If $m(\eta, x^*) = \delta$ there is no liquidation of the long-term investment. If $m(\eta, x^*) > \delta$, early withdrawals equal $m(\eta, x^*)D^R(1 + r_2^D)$, which is satisfied by liquidating some of the risky project. In particular, let $0 \leq y \leq 1$ be the fraction of the risky project that is liquidated. Thus, for a given value of η , the amount of long-term investment that needs to be liquidated is given by

$$\begin{aligned} m(\eta, x^*)D^R(1 + r_2^D) &= LIQ_1 + \xi \cdot y(\eta, x^*) \cdot I \\ \Rightarrow y(\eta, x^*) &= \frac{m(\eta, x^*)D^R(1 + r_2^D) - LIQ_1}{\xi \cdot I}. \end{aligned} \quad (4)$$

Observe that for $m(\eta, x^*) = \delta$, $y(\eta, x^*) = 0$ and that for $m(\eta, x^*) = \theta$, $y(\eta, x^*) = 1$.

First, suppose that $\delta \leq m(\eta, x^*) \leq \theta = m(\eta^*, x^*)$, which means that the bank survives to period

3. A patient depositor who withdraws faces the following budget constraint in period 2,

$$P_{sec}x_{sec}^{R,no-run,withdraw} + LIQ_2^{R,no-run,withdraw} \leq P_{sec}x_{eq}^R + D^R(1 + r_2^D) + LIQ_1^R + e_2^R, \quad (5)$$

and in period 3,

$$c_{3s}^{R,no-run,withdraw} \leq x_{sec}^{R,no-run,withdraw} DPS_{3s} + LIQ_2^{R,no-run,withdraw}. \quad (6)$$

Similarly, the constraints for a patient depositor who waits are

$$P_{sec}x_{sec}^{R,no-run,wait} + LIQ_2^{R,no-run,wait} \leq P_{sec}x_{eq}^R + LIQ_1^R + e_2^R, \quad (7)$$

and in period 3,

$$c_{3s}^{R,no-run,wait} \leq x_{sec}^{R,no-run,wait} DPS_{3s} + V_{3s}^D D^R(1 + r_3^D) + LIQ_2^{R,no-run,wait}. \quad (8)$$

From market clearing, it follows that

$$(m(\eta, x^*) - \delta)x_{sec}^{R,no-run,withdraw} + (1 - m(\eta, x^*))x_{sec}^{R,no-run,wait} = x_{eq}^R. \quad (9)$$

The dividends per share (DPS) are given by

$$DPS_{3s} = \frac{1}{x_{eq}^R + EB} [(1 - y(\eta, x^*)) \cdot V_{3s}^I I(1 + r^I) + LIQ_2 - (1 - m(\eta, x^*))V_{3s}^D D^R(1 + r_3^D)], \quad (10)$$

where

$$V_{3s}^D = \min \left[1, \frac{(1 - y(\eta, x^*)) \cdot V_{3s}^I I(1 + r^I) + LIQ_2}{(1 - m(\eta, x^*))D^R(1 + r_3^D)} \right]. \quad (11)$$

For $\theta \leq m(\eta, x^*) \leq 1$, the bank is liquidated. With probability $\frac{\theta}{m(\eta, x^*)}$, a patient depositor that withdraws early receives

$$c_{3s}^{R,run,paid} \leq D^R(1 + r_2^D) + LIQ_1^R + e_2^R, \quad (12)$$

and with probability $1 - \frac{\theta}{m(\eta, x^*)}$, he gets

$$c_{3s}^{R,run,unpaid} \leq LIQ_1^R + e_2^R, \quad (13)$$

which is equal to the consumption of a patient depositor that does not withdraw.

We assume that the decision on whether or not to withdraw is taken before the secondary market for equity meets and, thus, before depositors have the opportunity to learn the true state η , since the secondary market would aggregate private information. Rochet and Vives (2004) make a similar assumption. See Atkeson (2000) for a general discussion and Angeletos and Werning (2006) for a coordination game with incomplete information where financial prices can transmit the dispersed private information.

Note that given that the bank is liquidated, there will be no trade and the price of equity is $P_{sec} = 0$. For $\delta \leq m(\eta, x^*) \leq \theta$, the price in the secondary market depends on the proportion of early withdrawals, which determine DPS_{3s} , and is given by

$$P_{sec} = \frac{\sum_s \omega_{3s} (z_1 + \eta) U^{R'}(c_{3s}^{R,no-run,wait}) DPS_{3s}}{\sum_s \omega_{3s} (z_1 + \eta) U^{R'}(c_{3s}^{R,no-run,wait})} = \frac{\sum_s \omega_{3s} (z_1 + \eta) U^{R'}(c_{3s}^{R,no-run,withdraw}) DPS_{3s}}{\sum_s \omega_{3s} (z_1 + \eta) U^{R'}(c_{3s}^{R,no-run,withdraw})}. \quad (14)$$

Together with the market clearing condition (9), this condition determines the equity purchases in the secondary equity market, $x_{sec}^{R,no-run,wait}$ and $x_{sec}^{R,no-run,withdraw}$, for a given level of early withdrawals $m(\eta, x^*)$ or fundamentals η .

The utility differential for a patient depositor between waiting and withdrawing as a function of the fundamental and the signal threshold is given by

$$v(\eta, m(\eta, x^*)) = \begin{cases} \sum_s \omega_{3s} (z_1 + \eta) \left[U^R(c_{3s}^{R,no-run,wait}) - U^R(c_{3s}^{R,no-run,withdraw}) \right] & \text{if } \delta \leq m(\eta, x^*) \leq \theta \\ \frac{\theta}{m(\eta, x^*)} \left[U^R(c_{3s}^{R,run,unpaid}) - U^R(c_{3s}^{R,run,paid}) \right] & \text{if } \theta \leq m(\eta, x^*) \leq 1 \end{cases}. \quad (15)$$

Define as $\Delta(x_i, x^*)$ the utility differential between waiting and withdrawing for threshold x^* and signal x_i . To compute $\Delta(x_i, x^*)$ note that since η and the error terms ε_i are uniformly distributed, the agent's posterior distribution of η is uniformly distributed over $[x_i - \varepsilon_i, x_i + \varepsilon_i]$. Thus, $\Delta(x_i, x^*)$

is simply the average of the expectation of $v(\eta, m(\eta, x^*))$ over m , $\mathbb{E}_m v(\eta, m(\eta, x^*))$, over this range, i.e.,

$$\begin{aligned}\Delta(x_i, x^*) &= \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} \mathbb{E}_m v(\eta, m(\eta, x^*)) d\eta \\ &= \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} \int_{m=\delta}^1 v(\eta, m(\eta, x^*)) dF_\eta(m) d\eta.\end{aligned}\quad (16)$$

For $\Delta(x_i, x^*) > 0$, a patient depositor will wait, while for $\Delta(x_i, x^*) < 0$ he will withdraw at $t = 2$. A patient depositor who receives signal x^* is indifferent between waiting and withdrawing, i.e., $\Delta(x^*, x^*) = 0$.

As η goes from $x^* - \varepsilon$ to $x^* + \varepsilon$, m decreases linearly from $\delta + (1 - \delta) \frac{x^* + \varepsilon - x^* + \varepsilon}{2\varepsilon} = 1$ to $\delta + (1 - \delta) \frac{x^* + \varepsilon - x^* - \varepsilon}{2\varepsilon} = \delta$, because both the fundamentals and noise are uniformly distributed. Also, inverting equation (2), we get $\eta = x^* + \varepsilon \left(1 - 2 \frac{m - \delta}{1 - \delta}\right)$. Thus, $\Delta(x^*, x^*) = 0$ implies that:

$$\begin{aligned}& \int_{m=\delta}^{\theta} \sum_s \omega_{3s} \left(z_1 + x^* + \varepsilon \left(1 - 2 \frac{m - \delta}{1 - \delta}\right) \right) U^R(c_{3s}^{R, no-run, wait}) dm + \int_{m=\theta}^1 \frac{\theta}{m} U^R(c_{3s}^{R, run, unpaid}) dm = \\ & \int_{m=\delta}^{\theta} \sum_s \omega_{3s} \left(z_1 + x^* + \varepsilon \left(1 - 2 \frac{m - \delta}{1 - \delta}\right) \right) U^R(c_{3s}^{R, no-run, withdraw}) dm + \int_{m=\theta}^1 \frac{\theta}{m} U^R(c_{3s}^{R, run, paid}) dm.\end{aligned}\quad (17)$$

Solving equation (17) yields the signal threshold x^* as a function of η^* . Finally, the simultaneous solution of (3) and (17) determines x^* and η^* . At the limit when noise collapses to zero, i.e., $\varepsilon \rightarrow 0$, $\eta^* \rightarrow x^*$ from equation (3).

The probability of a bank run is thus computed as $q = \text{Prob}(\eta \leq \eta^*) = \frac{\eta^* + \bar{\eta}}{2\bar{\eta}}$. In order to compute η^* , equations (3) and (17) need to be solved simultaneously with all the equilibrium conditions defined in the paper plus equations (9) and (14) which yield the out-of-equilibrium trades in the secondary equity market. In our results in the paper, we assume that the probability of a run is a decreasing function of θ . If one believes that bank-runs are driven by payoff relevant variables rather than sunspots, it is hard to imagine a plausible model that would not have this property. By inspection one can tell that η^* will be a complicated function of most of the exogenous parameters in the economy, including ξ , and that θ will also be one of the factors that determines q . Hence, our assumption can be rationalized by appealing to a first-order Taylor approximation of the func-

tion that determines q . Finally, the functional form for ω_{3s} needs to be specified, which allow for additional flexibility in calibrating the probability of a bank-run in the initial equilibrium.

2 Existence and uniqueness of threshold equilibrium

$\Delta(x_i, x^*)$ is continuous in x_i because it is an integral in which the limits of integration are continuous in x_i and the integrand is bounded. From the intermediate value theorem, it suffices to show that there exist \underline{x}_i and \bar{x}_i such that $\Delta(\underline{x}_i, x^*) < 0$ and $\Delta(\bar{x}_i, x^*) > 0$. This is guaranteed if we assume the existence of lower (and upper) dominance region where fundamentals are bad (good) enough that every agents runs (or waits) independent of his belief concerning other patient agents' behavior.

Denote by η^{LD} the value of fundamentals such that

$\sum_s \omega_{3s}(z_1 + \eta^{LD}) \left[U^R(c_{3s}^{R,no-run,wait}) - U^R(c_{3s}^{R,no-run,withdraw}) \right] = 0$ for $m = \delta$ and refer to the interval $[-\bar{\eta}, \eta^{LD}]$ as the lower dominance region. Consider a realization of $\eta \geq -\bar{\eta}$ such that $\omega_{3b} = 1$. Then a patient agent will withdraw rather than wait if $U^R(c_{3s}^{R,no-run,wait}) < U^R(c_{3s}^{R,no-run,withdraw})$. Assume for simplicity that $LIQ_1^R = 0$ and that the bank holds enough liquidity to serve early withdrawals for impatient agents, i.e. $LIQ_1 = \delta \cdot D^R$. After substituting, patient agents will unambiguously run if $\frac{V_{3b}I(1+r^I)}{1-\delta} < D^R$ or $V_{3b}(1+r^I) < (1-\delta)(1+LR-CR)$. All variables in the previous inequality are predetermined. The inequality holds in the competitive equilibrium we examine in the paper, but we need to verify that it holds for all $\theta \leq 1$ to guarantee the existence of a lower dominance region when $q > 0$. We discuss the case of capital regulation, but the analysis is the same for other tools which reduce the probability of a bank run. As the capital ratio (CR) increases the liquidity ratio (LR) decreases and the left hand side of the previous inequality becomes smaller. Thus, we need to guarantee that it holds as θ approaches 1 or equivalently as CR and LR approach $1 - \xi$ and $\frac{\delta}{1-\delta}$ respectively. The inequality can then be written as $V_{3b}(1+r^I) < \xi$, which intuitively says that patient agents will unambiguously run if the long-term payoffs of the project in the bad state is lower than the early liquidation value. By continuity, the inequality will hold for higher values of η such that ω_{3b} is an open interval bounded above by 1, and there exists a lower dominance region defined by the interval $[-\bar{\eta}, \eta^{LD}]$ for small enough noise such that $\eta^{LD} > -\bar{\eta} + 2\epsilon$.

Regarding the upper dominance region $[\eta^{UD}, \bar{\eta}]$ with $\eta^{UD} < \bar{\eta} - 2\epsilon$, we need to modify the investment technology as is done in Goldstein and Pauzner (2005). In practice, this requires the liq-

liquidation value to be contingent on the state realization η , such that $\theta(\eta^{UD}) = \frac{LQ_1 + \xi(\eta^{UD})I}{D^R(1+r_2^D)} = 1$ and $\xi'(\eta) > 0$ for $\eta > \eta^{UD}$. In other words, when fundamentals are very strong the liquidation value of the bank is sufficient to serve all deposit withdrawals and a patient agent will unambiguously wait until the third period and receive a higher return on his deposits. Note that ξ can endogenously depend on the state realization if the model is extended to consider an outside investor buying the liquidated loans given his expectations about future delivery.

With respect to the uniqueness of the threshold equilibrium, observe that the second component of (15) is always negative and increasing in m as in Goldstein and Pauzner (2005). The first component is decreasing in m and crosses zero once, since for $m = \delta$ the expression is reduced to the incentive compatibility constraint when there is not a run, which is always satisfied, and for $m = \theta$ equity is worthless and all risky investment is liquidated to serve early withdrawals. Hence depositors who wait get nothing and the utility of the ones that withdraw is higher. To see that the first component in (15) is strictly decreasing, note that the depositors who wait will receive a lower repayment on deposits in the bankruptcy state as m increases (equation (11)). Thus, they will be less willing to purchase equity in the secondary market and would rather invest their wealth in the safe asset. Consequently, they will receive a lower income in the good (and potentially medium) state of the world compared to the depositors who withdraw early. The dividend payments will be lower for both, but the utility of the ones who wait will decrease by more. This can be seen from equation (14). The marginal utility of depositors who wait increases in the bad state as m increases. This requires a bigger increase in the marginal utility in the good/medium state for depositors who wait than for those who withdraw. Given that the income for the former is higher than for the latter (otherwise a run would occur all the time) and due to the concavity of the utility function, the utility of the depositors who wait will decrease more than the utility of the ones who withdraw as m increases. Hence, there are one-sided strategic complementarities, i.e., v is decreasing whenever it is positive, and the proof of Goldstein and Pauzner (2005) goes through.

References

Angeletos, G-M. and I. Werning (2006), 'Crises and prices: Information aggregation, multiplicity and volatility', *American Economic Review* **96**(5), 1720–1736.

- Atkeson, A. (2000), 'Rethinking multiple equilibria in macroeconomic modeling: Comment', *NBER Macroeconomics Annual* **15**, 162–171.
- Carlsson, H. and E. van Damme (1993), 'Global games and equilibrium selection', *Econometrica* **61**(5), 989–1018.
- Goldstein, I. and A. Pauzner (2005), 'Demand-deposit contracts and the probability of bank runs', *Journal of Finance* **60**(3), 1293–1327.
- Morris, S. and H.S. Shin (1998), 'Unique equilibrium in a model of self-fulfilling currency attacks', *American Economic Review* **88**(3), 587–597.
- Rochet, J-C. and X. Vives (2004), 'Coordination failures and the lender of last resort: Was Bagehot right after all?', *Journal of the European Economic Association* **2**(6), 1116–1147.