Appendix 1: Equations for small open economy.

Appendix Table 1

Equations	Variables
1. Households	$C \quad w \quad L$
1) $C^{-\rho} w = \kappa L^{\psi}$	$r \quad \tilde{d} \quad \tilde{q}$
2) $\beta(1-\lambda) = \frac{1}{1+r}$	
3) $\tilde{q}(\tilde{z}) = \frac{\beta(1-\lambda)}{(1-\beta(1-\lambda))} \tilde{d}(\tilde{z})$	
2. Final Goods	$\tilde{Y}^{xx} \quad P^{*} \quad \tilde{P}^{xx} \quad Y^{*}$
4) $\tilde{Y}^{xx} = \left(\frac{\tilde{P}^{xx}}{P^{*}}\right)^{-\sigma} Y^{*}$,	$\tilde{Y}^{nx} \quad \tilde{P}^{nx} \quad Y \quad P$
5) $\tilde{Y}^{nx} = \left(\frac{\tilde{P}^{nx}}{P}\right)^{-\sigma} Y$,	$\tilde{Y}^{xd} \quad \tilde{P}^{xd}$
6) $\tilde{Y}^{xd} = \left(\frac{\tilde{P}^{xd}}{P}\right)^{-\sigma} Y$,	$\tilde{Y}^{x*} \quad p^{x*}$
7) $\tilde{Y}^{x*} = \left(\frac{p^{x*}}{P}\right)^{-\sigma} Y$	$\tilde{P}^{d} \quad N^{x} \quad N^{nx} \quad N$
8) $P^{1-\sigma} = \frac{N^{x}}{1\text{-}\lambda}\left(\tilde{P}^{xd}\right)^{1-\sigma} + \frac{N^{nx}}{1\text{-}\lambda}\left(\tilde{P}^{nx}\right)^{1-\sigma} + \left(p^{x^{*}}\right)^{1-\sigma}$	
9) $N\left(\tilde{P}^{d}\right)^{1-\sigma} = N^{x}\left(\tilde{P}^{xd}\right)^{1-\sigma} + N^{nx}\left(\tilde{P}^{nx}\right)^{1-\sigma}$	
3. Intermediate	
3.1 Average Nov-exporters	
10) $\tilde{V}^{nx} = \tilde{d}^{nx} + \tilde{q}^{nx}$	$\tilde{V}^{nx} \quad \tilde{d}^{nx} \quad q_{t}^{nx}$
11) $\tilde{q}^{nx} = \frac{\beta(1-\lambda)}{(1-\beta(1-\lambda))} \tilde{d}^{nx}$	$\tilde{L}_{t}^{nx} \quad \tilde{b}_{t}^{nx}$
	$\tilde{z}^{nx} \quad \tilde{\mu} \quad R \quad z_{x} \quad z_{d}$

12) $$\tilde{b}^{nx} = \frac{R}{R-1}\left(\frac{\tilde{P}^{nx}\tilde{Y}^{nx}}{P} - w\,\tilde{L}^{nx} - w\,f^{d} - \tilde{d}^{nx} \right)$$

13) $$R = 1 + r\,(1-\tau)$$

14) $$\tilde{l}^{nx} = \frac{\tilde{y}^{nx}}{A\tilde{z}^{nx}}$$

15) $$\xi\,\tilde{q}^{nx} = \phi^{d} w \left(\tilde{L}^{nx}(z_i) + f^{d} \right)$$

16) $$\tilde{\mu} = \frac{1/R - m}{\xi m}$$

17) $$\frac{\tilde{P}^{nx}}{P} = \frac{\sigma}{\sigma-1}\frac{w}{A\,\tilde{z}^{nx}}\left(1 + \phi^{d}\tilde{\mu}\right)$$

18) $$\left(\tilde{z}^{nx}\right)^{\sigma-1} = \frac{\theta\left(z_X^{\ \sigma-\theta-1} - z_D^{\ \sigma-\theta-1} \right)}{(\sigma-\theta-1)\left(z_D^{\ -\theta} - z_X^{\ -\theta} \right)}$$

3.2Average exporter

$\tilde{V}^{X}$ $\tilde{d}^{X}$ $\tilde{q}^{X}$

$\tilde{L}^{xd}$ $\tilde{L}^{xx}$ $\tilde{b}^{x}$

$\tilde{z}^{x}$ $\tilde{L}^{x}$

19) $$\tilde{V}^{x} = \tilde{d}^{x} + \tilde{q}^{x}$$

20) $$\tilde{q}^{x} = \frac{\beta\,(1-\lambda)}{\left(1-\beta\,(1-\lambda)\right)}\tilde{d}^{x}$$

21) $$\tilde{b}^{x} = \frac{R}{R-1}\left[\frac{\tilde{P}^{xd}\tilde{Y}^{xd}}{P} + \frac{\tilde{P}^{xx}\tilde{Y}^{xx}}{P} - w\,\left(\tilde{L}^{xd} + \tilde{L}^{xx}\right) - w\,\left(f^{d} + f^{x}\right) - \tilde{d}\right.$$

22) $$\tilde{l}^{xd} = \frac{\tilde{y}^{xd}}{A\tilde{z}^{x}}$$

23) $$\tilde{l}^{xx} = \frac{\tilde{y}^{xx}}{A\tilde{z}^{x}\left(1\text{-}\tau_x\right)}$$

24) $$\tilde{L}^{x} = \tilde{L}^{xd} + \tilde{L}^{xx}$$

25) $$\xi\tilde{q}^{x} + \gamma^{xx}\frac{\tilde{p}^{xx}\tilde{y}^{xx}}{P} = \phi^{d}\left(w\tilde{l}^{xd} + w\,f^{d}\right) + \phi^{x}\left(w\tilde{l}^{xx} + w\,f^{x}\right)$$

26) $\frac{\tilde{P}^{xd}}{P}=\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}^{x}}\left(1+\phi^{d}\tilde{\mu}\right)$	
27) $\left(\tilde{z}^{x}\right)^{\sigma-1}=\frac{-\theta z_{x}^{\ \sigma-\theta-1}}{\left(\sigma-\theta-1\right)z_{x}^{\ -\theta}}=\frac{-\theta z_{x}^{\ \sigma-1}}{\sigma-\theta-1}$	
28) $\frac{\tilde{P}^{xx}}{P}=\frac{\sigma}{\sigma-1}\frac{w_{t}}{A\,\tilde{z}^{x}\left(1-\tau_{x}\right)}\frac{1+\phi^{x}\tilde{\mu}}{1+\gamma^{xx}\tilde{\mu}}$	
3.3Marginal New Entrant	
29) $V^{nx}\left(z_{d}\right)=d_{d}^{nx}+q_{d}^{nx}$	$V_{d}^{nx}\ d_{d}^{nx}\ q_{d}^{nx}$
30) $b_{d}^{nx}=\frac{R}{R-1}\left[\frac{p_{d}^{nx}y_{d}^{nx}}{p}-w\ \left(l_{d}^{nx}+f^{d}\right)-d_{d}^{nx}\right]$	$l_{d}^{nx}\ b_{d}^{nx}\ y_{d}^{nx}\ p_{d}^{nx}$
31) $l_{d}^{nx}=\frac{y_{d}^{nx}}{A\,z_{d}}$	
32) $y_{d}^{nx}=\left(\frac{p_{d}^{nx}}{P}\right)^{-\sigma}Y$	
33) $q_{d}^{nx}=\frac{\beta\ \left(1-\lambda\right)}{\left(1-\beta\ \left(1-\lambda\right)\right)}d_{d}^{nx}$	
34) $\xi q_{d}^{nx}=\phi^{d}w\left(l_{d}^{nx}+f^{d}\right)$	
35) $\frac{p_{d}^{nx}}{P}=\frac{\sigma}{\sigma-1}\frac{w}{A\,z_{d}}\left(1+\phi^{d}\mu\ \right),$	
36) $\frac{p_{d}^{nx}y_{d}^{nx}}{P}-w\ \left(l_{d}^{nx}+f^{d}\right)-\left(b_{d}^{nx}-\frac{b_{d}^{nx}}{R}\right)+q_{d}^{nx}-K^{E}=0$	
3.4 marginal exporter	$V_{x}^{x}\ d_{x}^{x}\ q_{x}^{x}$
37) $V^{x}\left(z_{x}\right)=d_{x}^{x}+q_{x}^{x}$	$p_{x}^{xd}\ y_{x}^{xd}\ p_{x}^{xx}\ y_{x}^{xx}$
38) $b_{x}^{x}=\frac{R}{R-1}\left[\frac{p_{x}^{xd}y_{x}^{xd}}{P}+\frac{p_{x}^{xx}y_{x}^{xx}}{P}-w\ \left(l_{x}^{xd}+l_{x}^{xx}\right)-w\ \left(f^{d}+f^{x}\right)-d_{x}^{x}\right]$	$b_{x}^{x}\ l_{x}^{xd}\ l_{x}^{xx}$

39) $l_x^{xd} = \frac{y_x^{xd}}{A\,z_x}$	$V_x^{nx}\ d_x^{nx}\ q_x^{nx}$ $p_x^{nx}\ y_x^{nx}\ l_x^{nx}\ b_x^{nx}$
40) $l_x^{xx} = \frac{y_x^{xx}}{A\,z_x\left(1-\tau_x\right)}$	
41) $y_x^{xd} = \left(\frac{p_x^{xd}}{P}\right)^{-\sigma} Y$	
42) $y_x^{xx}\left(z_i\right) = \left(\frac{p_{xt}^{xx}}{P^*}\right)^{-\sigma} Y^* = \left(\frac{p_x^{xx}}{P}\right)^{-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma} Y^*$	
43) $\xi q_x^x + \gamma^{xx}\frac{p_x^{xx} y_x^{xx}}{P} = \phi^d w\left(l_x^{xd} + f^d\right) + \phi^x w\left(l_x^{xx} + f^x\right)$	
44) $\frac{p_x^{xd}}{P} = \frac{\sigma}{\sigma-1}\frac{w}{A\,z_x}\left(1+\phi^d\mu\right)$	
45) $\frac{p_x^{xx}}{P} = \frac{\sigma}{\sigma-1}\frac{w}{A\,z_x}\frac{1+\phi^x\mu}{\left(1\text{-}\tau_x\right)\left(1+\gamma^{xx}\mu\right)}$	
46) $\frac{p_x^{nx} y_x^{nx}}{P_t} - w\left(l_x^{nx} + f^d\right) - \left(b_x^{nx} - \frac{b_x^{nx}}{R}\right) + q_x^{nx} - K^E$ $= \frac{p_x^{xd} y_x^{xd}}{P} + \frac{p_x^{xx} y_x^{xx}}{P} - w\left(l_x^{xd} + l_x^{xx} + f^d + f^d\right) - \left(b_x^{nx} - \frac{b_x^x}{R}\right) + q_x^x - \left(K^E + K^{EX}\right)$	
or: $\left(\frac{p_x^{xx} y_x^{xx}}{P} - w\,l_x^{xx} - w\,f^x\right) + \frac{b_x^x - b_x^{nx}}{R} + \left(q_x^x - q_x^{nx}\right) = K^{EX}$	
47) $q_x^x = \frac{\beta\left(1-\lambda\right)}{\left(1-\beta\left(1-\lambda\right)\right)} d_x^x$	
48) $q_x^{nx} = \frac{\beta\left(1-\lambda\right)}{\left(1-\beta\left(1-\lambda\right)\right)} d_x^{nx}$	
49) $\frac{p_x^{nx}}{P} = \frac{\sigma}{\sigma-1}\frac{w}{A\,z_x}\left(1+\phi^d\mu\right)$	$Ne\ \ Ne^x$
50) $d_x^{nx} = \frac{p_x^{nx} y_x^{nx}}{P} - w\,l_x^{nx} - b_x^{nx} + \frac{b_x^{nx}}{R} - w\,f^d$	

51) $V_x^{nx} = d_x^{nx} + q_x^{nx}$

52) $l_x^{nx} = \dfrac{y_x^{nx}}{A\, z_x}$

53) $y_x^{nx} = \left(\dfrac{p_x^{nx}}{P} \right)^{-\sigma} Y$

54) $\xi q_x^{nx} = \phi^d w \left(l_x^{nx} + f^d \right)$

4. Aggregation

55) $N^x \tilde{p}^{xx} \tilde{y}^{xx} = P^{X^*} Y^{X^*}$

56) $Y = Ne\, K^E + Ne^x K^{EX} + C$

57) $L = \dfrac{N^{nx} \tilde{L}^{nx} + N^x \left(\tilde{L}^{xd} + \tilde{L}^{xx} \right) + \left(N^{nx} + N^x \right) f^d + N^x f^x}{1-\lambda}$

58) $N = N^{nx} + N^x$

59) $N = (1\text{-}\lambda)(N + Ne)$

60) $N = (1\text{-}\lambda)(1\text{-}G(z_d))M = (1\text{-}\lambda) z_D^{-\theta}$

61) $N^x = (1\text{-}\lambda)(N^x + Ne^x)$

62) $N^x = (1\text{-}\lambda)(1\text{-}G(z_x))M = (1\text{-}\lambda) M\, z_x^{-\theta}$

63) $N\, \tilde{d} = N^{nx} \tilde{d}^{nx} + N^x \tilde{d}^x$

64) $P^* = 0.51$

65) $p^{x*} = 1$

66) $Y^* = 5Y$

Appendix 2: Analytical Results for Other Variables

Non-exporters

(1) Sales

Combining the pricing equation (19) and the market demand, Eq. (8), we have

$$sales = \frac{p^{nx}(z_i)}{P} y^{nx}(z_i) = \left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\left(1+\phi^d\mu\right)\right)^{1-\sigma} Y \cdot$$

Taking the first-order derivative and using the first-order derivative of equity price w.r.t. to z_i, that is , Eq. (36), give

$$\frac{\partial sales^{nx}(z_i)}{\partial z_i} = \frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{\xi}{\phi^d}\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i}.$$

It shows that, first, sales is increasing in productivity when $\sigma > 1$, which is also the condition for equity price to increase in productivity; second, the marginal effect of productivity on firm sale for a non-exporter is linear in the marginal effect of productivity on its equity price.

In particular we find a linear relationship between the log level of sales and the log level of productivity,

$$\log\left(sales^{nx}\right) = \left(1-\sigma\right)\log\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\left(1+\phi^d\mu\right)\right) + \log\left(Y\right) + \left(\sigma-1\right)\log\left(z_i\right),$$

which implies that

$$\frac{\partial \log\left(sales^{nx}\left(z_i\right)\right)}{\partial \log\left(z_i\right)} = \left(\sigma-1\right),$$

given the aggregate market condition.

(2) Sales-equity ratio

In section 4.1, Eq. (38) shows that the marginal effect of z_i on the long-term debt-equity ratio essentially relies on the sales-equity ratio. So here we show how changing productivity affects the sales-equity ratio formally.

Taking the first-order derivative of the sales-equity ratio w.r.t. z_i, we have

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i} = \frac{1}{q^{nx}(z_i)}\frac{\partial sales^{nx}(z_i)}{\partial z_i} - \frac{sales^{nx}(z_i)}{\left(q^{nx}(z_i)\right)^2}\frac{\partial q^{nx}(z_i)}{\partial z_i},$$

or

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i} = \frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{1}{\left(q^{nx}(z_i)\right)^2}\left(\frac{\frac{\partial sales^{nx}(z_i)}{\partial z_i}}{\frac{\partial q^{nx}(z_i)}{\partial z_i}} q^{nx}(z_i) - sales^{nx}(z_i)\right).$$

Given the marginal effect of productivity on equity price, Eq. (36), we have the following equation,

$$sales^{nx}(z_i)=\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{\xi}{\phi^d}\frac{z_i}{\sigma-1}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right),$$

which, combining with the marginal effect of productivity on sales,

$$\frac{\partial sales^{nx}(z_i)}{\partial z_i}=\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{\xi}{\phi^d}\frac{\partial q^{nx}(z_i)}{\partial z_i},$$

gives

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i}=\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{1}{\left(q^{nx}(z_i)\right)^2}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{\xi}{\phi^d}\left(q^{nx}(z_i)-\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{z_i}{\sigma-1}\right).$$

Using the enforcement constraint, Eq. (14),

$$q^{nx}(z_i)=\frac{\phi^d}{\xi_q}\left(\frac{\sigma}{\sigma-1}\left(\frac{1+\phi^d\mu_t}{1+\gamma_t^{nx}\mu_t}\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}+\frac{\phi^d wf^D}{\xi_q}$$

and the marginal effect of productivity on equity price, Eq. (36),

$$\frac{\partial q^{nx}(z_i)}{\partial z_i}=\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma-1}{z_i}\right),$$

we have

$$q^{nx}(z_i)-\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{z_i}{\sigma-1}=\left(\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}+\frac{\phi^d wf^D}{\xi}\right)-\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}=\frac{\phi}{\xi}wf^D$$

Hence,

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i}=\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{1}{\left(q^{nx}(z_i)\right)^2}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)wf^D.$$

(3) Product quantity

Combining the pricing equation (19) and the market demand, Eq. (8), we have

$$y^{nx}(z_i)=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y.$$

This implies

$$\frac{\partial y^{nx}(z_i)}{\partial z_i}=\frac{Az_i}{w}\frac{\sigma}{\sigma-1}\frac{\xi_q}{\phi}\frac{\partial q^{nx}(z_i)}{\partial z_i}.$$

A similar linearity is found in the relation between the log level of product quantity and the log level of productivity, given by

$$\log\left(y^{nx}(z_i)\right)=\log\left(\left(\frac{\sigma}{\sigma-1}\left(\frac{1+\phi^d\mu_t}{1+\gamma_t^{nx}\mu_t}\right)\frac{w}{A}\right)^{-\sigma}Y\right)+\sigma\log\left(z_i\right)$$

which implies

$$\frac{\partial\log\left(y^{nx}(z_i)\right)}{\partial\log\left(z_i\right)}=\sigma.$$

(4) Profit

Given $\pi^{nx}(z_i)=\frac{p^{nx}(z_i)}{P}y^{nx}(z_i)-\left(wl^{nx}(z_i)+wf^{nx}\right)$, and combining the pricing equation (19) and the market demand, Eq. (8), and the enforcement constraint, Eq. (14), we have

$$\pi^{nx}(z_i)=sale^{nx}(z_i)-\frac{\xi_q q^{nx}(z_i)}{\phi^d}.$$

Taking the first-order derivative thus gives

$$\frac{\partial \pi^{nx}(z_i)}{\partial z_i}=\frac{\partial q^{nx}}{\partial z_i}\frac{\xi}{\phi^d}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)-1\right).$$

The first item in the bracket captures the rising sales effect associated with the rising z_i, while the second item captures the rising production cost associated with rising productivity.

(5) Firm value

Given that $V^{nx}(z_i)=q^{nx}(z_i)+d^{nx}(z_i)=\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^{nx}(z_i)$, we have

$$\frac{\partial V^{nx}(z_i)}{\partial z_i}=\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\frac{\partial q^{nx}(z_i)}{\partial z_i}.$$

Exporters

The equations for exporters are analogous, but with additional parameters government the working capital needs of exporters, and the collateral value of exporter accounts receivable.

(1) Sales

Given that $sales^{x}(z_i)=sales^{xd}(z_i)+sales^{xx}(z_i)$, $sales^{xd}(z_i)=\frac{p^{xd}(z_i)}{P}y^{xd}(z_i)$, and $sales^{xx}(z_i)=\frac{p^{xx}(z_i)}{P}y^{xx}(z_i)$, we have

$$\frac{\partial sales^{xd}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{1-\sigma}Y,$$

$$\frac{\partial sales^{xx}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*,$$

$$\frac{\partial sales^{x}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\left(1+\phi^d\mu\right)^{1-\sigma}Y_t+\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right).$$

Note, the presence of the asymmetric working capital needs ($\phi^d\neq\phi^x$) and the reliance on trade credit ($\gamma^{xx} \neq 0$) does not allow us to represent $\frac{\partial sales^{x}(z_i)}{\partial z_i}$ as a function of $\frac{\partial q^{x}(z_i)}{\partial z_i}$, as we did for the non-exporter.

As for the non-exporter, we find a linear relationship between the log level of sales and the log level of productivity,

$$\log\left(sales^{x}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w}{A}(1+\mu)\right)+\log\left(\left(1+\phi^{d}\mu\right)^{1-\sigma}Y+\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\right)+(\sigma-1)\log\left(z_i\right),$$

$$\log\left(sales^{xd}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\left(1+\phi^{d}\mu\right)\right)+\log(Y)+(\sigma-1)\log\left(z_i\right),$$

$$\log\left(sales^{xx}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t}\right)+\log\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\right)+(\sigma-1)\log\left(z_i\right),$$

which implies that $\frac{\partial\log\left(sales^{xd}(z_i)\right)}{\partial\log(z_i)}=\frac{\partial\log\left(sales^{xx}(z_i)\right)}{\partial\log(z_i)}=\frac{\partial\log\left(sales^{x}(z_i)\right)}{\partial\log(z_i)}=(\sigma-1)$.

(2) Sales-equity ratio

In section 4.1, Eq. (41) shows that the marginal effect of z_i on the long-term debt-equity ratio essentially relies on the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}$, and the export sales-equity ratio. So here we show how changing productivity affects these ratios formally.

The domestic and export sales-equity ratios are respectively given by

$$\frac{sales^{xd}(z_i)}{q^{x}(z_i)}=\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}\left(1+\phi^{d}\mu\right)^{1-\sigma}Y\frac{(z_i)^{\sigma-1}}{q^{x}(z_i)},$$

and

$$\frac{sales^{xx}(z_i)}{q^{x}(z_i)}=\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\frac{(z_i)^{\sigma-1}}{q^{x}(z_i)}.$$

Taking the first-order derivative of these ratios w.r.t. z_i, we have

$$\frac{\partial\left(\frac{sales^{xd}(z_i)}{q^{x}(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^{x}(z_i)^{2}}\left(\phi^{d}f^{d}+\phi^{x}f^{x}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\left(1+\phi^{d}\mu\right)^{1-\sigma}Y,$$

and

$$\frac{\partial\left(\frac{sales^{xx}(z_i)}{q^{x}(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^{x}(z_i)^{2}}\left(\phi^{d}f^{d}+\phi^{x}f^{x}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}.$$

The marginal effect of z_i on the total export sales-equity ratio is given correspondingly by

$$\frac{\partial\left(\frac{sales^x(z_i)}{q^x(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^x(z_i)^2}\frac{\sigma-1}{z_i}\left(\phi^d f^D+\phi^x f^X\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\left(\left(1+\phi^d\mu\right)^{1-\sigma}Y+\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right).$$

Again, we see the presence of the asymmetric working capital needs ($\phi^d \neq \phi^x$) and the reliance on trade credit ($\gamma^{xx} \neq 0$) prevents us from representing $\frac{\partial sales^x(z_i)}{\partial z_i}$ as a function of $\frac{\partial q^x(z_i)}{\partial z_i}$, as we did for the non-exporter.

(3) Product quantity

Combining the pricing equations (20)-(21) and the market demand, Eqs. (9) and (11), we have

$$\begin{aligned}y^x(z_i)&=y^{xd}(z_i)+\frac{1}{1\text{-}\tau_x}y^{xx}(z_i)\\&=\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)\right)^{-\sigma}Y_t+\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\end{aligned}.$$

This implies

$$\frac{\partial y^x(z_i)}{\partial z_i}=\frac{\sigma}{z_i}\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{-\sigma}\left(\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)^{-\sigma}Y_t+\left(\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\right).$$

Further, we have

$$\log\left(y^x(z_i)\right)=(-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t}\right)+\log\left(\begin{array}{l}\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)^{-\sigma}Y_t\\+\left(\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\end{array}\right)+\sigma\log(z_i)$$

and

$$\frac{\partial\log\left(y^x(z_i)\right)}{\partial\log(z_i)}=\sigma.$$

(4) Profit

Given $\pi^X(z_i)=\frac{p^D(z_i)}{P}y^D(z_i)+\frac{p^X(z_i)}{P}y^X(z_i)(1\text{-}\tau_x)-w\left(l^D(z_i)+l^X(z_i)+f^D+f^X\right)$,and combining the pricing equations (20)-(21) and the market demand, Eqs. (9) and (11), we have

$$\frac{\partial\pi^x(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y+\left(1-\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right)$$

and

$$\frac{\partial\left(\frac{\partial \pi^x(z_i)}{\partial z_i}\right)}{\partial \gamma^{xx}} = \frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1+\phi^x\mu}{(1-\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*(\sigma-1)\mu\frac{\left(\phi^x-\gamma^{xx}\right)\mu}{\left(1+\gamma^{xx}\mu\right)\left(1+\phi^x\mu\right)}.$$

(5) Firm value

Given that $V^x(z_i) = q^x(z_i) + d^x(z_i) = \left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^x(z_i)$, we have

$$\frac{\partial V^x(z_i)}{\partial z_i} = \left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\frac{\partial q^x(z_i)}{\partial z_i}$$

(6) Equity price

From the enforcement constraint, for an individual exporter i, we have that

$$\xi q^x(z_j) = \phi^d\left(wl^{xd}(z_j) + wf^D\right) + \phi^x\left(wl^{xx}(z_j) + wf^X\right) - \gamma^{xx}\frac{p^{xx}(z_j)y^{xx}(z_j)}{P}.$$

Combing the production function, Eq. (15), the pricing equations (20)-(21), and the market demand, Eqs. (9) and (11), and taking the derivatives of the equity price with respect to firm productivity gives that

$$\frac{\partial q^X(z_i)}{\partial z_i} = \frac{1}{\xi}\left(\frac{\sigma-1}{z_i}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\left(\begin{array}{l}\left(\phi^d\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\ +\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1-\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

and

$$\frac{\partial^2 q^X(z_i)}{\partial (z_i)^2} = \frac{1}{\xi}(\sigma-1)(\sigma-2)\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}(z_i)^{\sigma-3}\left(\begin{array}{l}\left(\phi^d\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\ +\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1-\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

Hence, $\frac{\partial q^x(z_i)}{\partial z_i} > 0$ if $\sigma > 1$, and further $\frac{\partial^2 q^x(z_i)}{\partial (z_i)^2} > 0$ if $\sigma > 2$. We also need the condition that $\gamma^{xx} < \phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$. As for non-exporters, when the conditions are satisfied, rising firm productivity will increase firm equity value more for larger exporters than for smaller exporters.

For exporters, the impact of productivity on equity prices works through three channels, the production for domestic sales, the production for export sales and the accounts receivable used as collateral. The first two channels raise firm equity value while the third one reduces it. This is because rising firm productivity is associated with rising sales in both domestic and foreign markets, and hence raises firm equity value. However, rising export sales provide a second type of collateral, that is, the accounts receivable, which reduces firms' reliance on equity as collateral.

Further, we have that

$$\frac{\partial\left(\frac{\partial q^{X}(z_i)}{\partial z_i}\right)}{\partial \gamma^{xx}} = \frac{1}{\xi}\left(\frac{\sigma-1}{z_i}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}(1\text{-}\tau_x)^{\sigma-1}(\sigma-1)\mu\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma-1}\frac{1}{1+\gamma^{xx}\mu}\left(\phi^x\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{P}{P^*}\right)^{-\sigma}Y^*. \quad \text{(A1)}$$

Eq. (37) shows that the rising reliance on accounts receivable as collateral (rising γ^{xx}) will amplify the effect of productivity on firm equity value, because $\frac{\partial\left(\frac{\partial q^{X}(z_i)}{\partial z_i}\right)}{\partial \gamma^{xx}} > 0$ when $\gamma^{xx} < \phi^x\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$ which is guaranteed by the condition that $\gamma^{xx} < \phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$.

(7) Bond position

From the dividend equation, (17), we have that

$$b^x(z_i) = \frac{R}{R-1}\left(\frac{p^{xd}(z_i)}{P}y^{xd}(z_i) + \frac{p^{xx}(z_i)}{P}y^{xx}(z_i)(1\text{-}\tau_x) - w\left(l^{xd}(z_i)+l^{xx}(z_i)+f^d+f^x\right) - d^x(z_i)\right).$$

Substituting the production function, Eq. (15), the pricing equations (20)-(21), the market demand equations (9)-(11), the firm value function, Eq. (18), , we have that

$$b^x(z_i) = \frac{R}{R-1}\left(\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\frac{p^{xd}(z_i)}{P}y^{xd}(z_i) + \left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\frac{p^{xx}(z_j)y^{xx}(z_j)}{P_t} - w\left(f^d+f^x\right) - \left(\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^x(z_i)\right). \quad \text{(A2)}$$

Note, the presence of trade credit and the asymmetric working capital requirements for exports and for domestic products in the enforcement constraint, prevent us from using the enforcement constraint to substitute the production cost with firm equity value, as we did for the non-exporters.

Then taking the derivatives of bond position with respect to firm productivity, by combining the pricing equations (20)-(21), and the market demand equations (9)-(11), yields

$$\frac{\partial b^x(z_i)}{\partial z_i} = \frac{R}{R-1}\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\begin{array}{l}\left(\underbrace{1}_{dom\_sales} - \underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}}_{dom\_var\_cost} - \underbrace{\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}}_{equiy\_Needs\_dom\_cost}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y \\ +\left(\underbrace{1}_{exp\_sales} - \underbrace{\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}}_{exp\_var\_cost} - \underbrace{\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)}_{equity\_needs\_exp\_cost}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right).$$

and

$$\frac{\partial^2 b^x(z_i)}{\partial (z_i)^2} = \frac{R}{R-1}(\sigma-1)(\sigma-2)\left(\frac{w}{A}\right)^{1-\sigma}(z_i)^{\sigma-3}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\begin{array}{l}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu} - \frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y \\ +\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu} - \frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

Further, we notice that

$$\frac{\partial\left(\frac{\partial b^x(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}=\frac{R}{R-1}\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\left(\frac{1+\phi^x\mu}{1\text{-}\tau_x}\right)^{1-\sigma}\left(1+\gamma^{xx}\mu\right)^{\sigma\text{-}2}\begin{pmatrix}(\sigma\text{-}1)\mu-(\sigma-1)\frac{\mu\left(1+\gamma^{xx}\mu\right)}{1+\phi^x\mu}\\-\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left((\sigma-1)\frac{\phi^x}{1+\phi^x\mu}\mu\left(1+\gamma^{xx}\mu\right)-\left(1+\gamma^{xx}\mu\right)-\gamma^{xx}(\sigma\text{-}1)\mu\right)\end{pmatrix}.$$

However, the complicated representations of the first- and second-order derivatives make us difficult to conclude the conditions that ensure $\frac{\partial b^x(z_i)}{\partial z_i}>0$ or $\frac{\partial^2 b^x(z_i)}{\partial\left(z_i\right)^2}>0$ or $\frac{\partial\left(\frac{\partial b^x(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}>0$.

(8) Long-term Debt-to-equity ratio

Now we are ready to look at the leverage ratio defined in Eq. (34), which relies on two ratios, the long-term debt-equity ratio and the short-term debt equity ratio. Let $LR_1^x(z_i)$ and $SR_1^x(z_i)$ denote the long-term and short-term ratios respectively, thus $LR_1^x(z_j)=\frac{b^x(z_j)}{q^{nx}\left(z_j\right)}$, and $SR_1^x(z_i)=\frac{IntraLoan\left(z_i\right)}{q\left(z_i\right)}$.

Using Eq. (40), we have the long-term debt-equity ratio

$$LR^x(z_i)=\frac{R}{R-1}\left[\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\frac{sales^{xx}(z_i)}{q^x(z_i)}-\frac{w\left(f^d+f^x\right)}{q^x(z_i)}-\left(\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\right], \quad \text{(A3)}$$

which shows the effect of changing z_i on the long-term debt-equity ratio is through the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}$, the export sales-equity ratio, $\frac{sales^{xx}(z_i)}{q^x(z_i)}$, and the fixed costs-equity ratio, $\frac{w\left(f^d+f^x\right)}{q^x(z_i)}$.

A few steps of calculations show that

$$\frac{\partial LR^x(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{1}{q^x\left(z_i\right)^2}\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{w}{\xi}\left[\begin{array}{l}\left(\phi^d f^D+\phi^x f^X\right)\begin{pmatrix}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{pmatrix}\\+\left(f^D+f^X\right)\begin{pmatrix}\phi^d\frac{\sigma-1}{\sigma}\left(1+\phi^d\mu\right)^{-\sigma}Y\\+\left(\phi^x\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{pmatrix}\end{array}\right]$$

Thus, $\frac{\partial LR^{nx}(z_i)}{\partial z_i}>0$ if $\sigma>1$ and $\frac{\gamma^{xx}}{\phi^x}<\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}<1$.

Additionally, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D=f^X=0$, the leverage ratio is the same for all exporters, regardless of productivity level.

Further, we have $\frac{\partial\left(\frac{\partial LR^x(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}>0$ if

$$\frac{\phi^x-\gamma^{xx}}{\left(1+\gamma^{xx}\mu\right)\left(1+\phi^x\mu\right)}\left(\left(\phi^d f^d+\phi^x f^x\right)\mu+\left(f^D+f^X\right)\right)>f^D+f^X.$$

(9) Short-term Debt-to-equity ratio

Given $SR^x(z_i)=\frac{IntraLoan^x(z_i)}{q^x(z_i)}$ and $IntraLoan(z_i)=\phi^d\left(w\,l^{xd}\left(z_j\right)+w\,f^D\right)+\phi^x\left(w\,l^{xx}\left(z_j\right)+w\,f^X\right)$,

and combining with the production function, Eq. (15), the pricing equations (20)-(21), the market demand equations (9)-(11), we have

$$IntraLoan^x\left(z_i\right)==\phi^d\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}sales_t^{xd}\left(z_j\right)+\phi^x\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}sales_t^{xx}\left(z_j\right)+\left(\phi^d w\,f^D+\phi^x w\,f^X\right)$$

and

$$SR^x\left(z_i\right)=\frac{\sigma-1}{\sigma}\frac{\phi^d}{1+\phi^d\mu}\frac{sales^{xd}\left(z_j\right)}{q^x\left(z_i\right)}+\phi^x\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\frac{sales^{xx}\left(z_j\right)}{q^x\left(z_i\right)}+\frac{\phi^d w\,f^D+\phi^x w\,f^X}{q^x\left(z_i\right)},$$

which shows the effect of changing z_i on the short-term debt-equity ratio is through the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^x(z_i)}$, the export sales-equity ratio, $\frac{sales^{xx}(z_i)}{q^x(z_i)}$, and the fixed costs-equity ratio, $\frac{w\left(f^d+f^x\right)}{q^x(z_i)}$, as well as the long-term debt-equity ratio.

A few steps of calculations show that

$$\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial z_i}=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{1}{q^x\left(z_i\right)^2}\frac{w}{\xi}\left(\phi^d f^d+\phi^x f^x\right)\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma\text{-}1}(1\text{-}\tau_x)^{\sigma\text{-}1}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\frac{\phi^x}{\sigma}+\gamma^{xx}\right)$$

Thus, $\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial z_i}>0$ if $\sigma>1$.

Additionally, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D=f^X=0$, the short-term leverage ratio is the same for all exporters, regardless of productivity level.

Further, we have

$$\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial\gamma^{xx}}=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{1}{q^X\left(z_i\right)^2}\frac{w}{\xi}\left(\phi^d f^d+\phi^x f^x\right)\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma\text{-}1}(1\text{-}\tau_x)^{\sigma\text{-}1}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\left(\frac{\phi^x\mu}{1+\phi^x\mu}+\frac{\sigma\gamma^{xx}+1}{1+\gamma^{xx}\mu}\right) \text{ and}$$

$\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial\gamma^{xx}}>0$ if $\sigma>1$.

(10) Intensive margin response to higher working capital requirement

The degree to which the intensive margin of exports is offset by capital restructuring can be understood by studying the price setting equation for exports (assuming for simplicity that export sales cannot be used as collateral (γ^{xx}=0)):

$$\frac{p^{xx}\left(z_i\right)}{P}=\frac{\sigma}{\sigma-1}\frac{w}{Az_i\left(1\text{-}\tau_x\right)}\left[1+\phi^x\mu^x\right].$$

The novel part of our equation is in square brackets, indicating an extra price markup due to the tightness of the collateral constraint. A rise in ϕ^x has two channels for raising export prices and hence reducing export sales. First there is a direct effect in that ϕ^x appears in the equation, and raises price by multiplying a given μ^x. So this effect is conditional on μ^x >0, meaning it requires the financial constraint is binding. This effect will be small, given that our steady state value of μ^x is small: $\mu^x = 0.224$ implies that the 100% rise in ϕ^x from 0.5 to 1 raises $\frac{p^{xx}\left(z_i\right)}{P}$ by only about 10% ($\frac{1+\phi^x\mu^x}{1+\phi^d\mu^x}$). This coincides with our numerical results that the rise in ϕ^x lowers export sales of a given firm z_i by about 8%, once one factors in the general equilibrium effects on wage and relative international price indexes.

Overall export sales of a firm z can be computed here as: $\frac{p^{xx}\left(z_x\right)y^{xx}\left(z_x\right)}{P}=\left(\frac{p^{xx}\left(z_i\right)}{P}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i\left(1\text{-}\tau_x\right)}\left(1+\phi^x\mu^x\right)\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*$. From our simulations, $\frac{p^{xx}\left(z_i\right)}{P}$ increases about 8.1765% but $\frac{P}{P^*}$ decreases about 3.811% for ϕ^x from 0.5 to 1. Here w drops about 1.71% so that $\frac{p^{xx}\left(z_i\right)}{P}$ does not increase by 10%. Given σ=3.8 and the tiny change in Y^* (-0.3060%), the impact on export sales: (1-3.8)*8.1765%-3.8*(-3.811%)=-8.411%.

The second effect is that, in the absence of capital structure, a rise in ϕ^x would make the exporter collateral constraint tighter, raising the value of μ^x. But given that our capital structure optimization implies the optimality condition that $\mu^k(z_i) = \frac{1/R - Em}{\xi Em}$, we know that under capital structure, the tightness of the collateral constraint does not move for different values of ϕ^x. So this second effect is completely eliminated by capital structure adjustments. The main lesson is that under endogenous capital structure, greater financial restrictions on working capitals need not have the effect that the past literature has assumed. Endogenous capital structure fundamentally changes this result, and can dramatically reduce effects on the intensive margin of export sales.