China's Model of Managing the Financial System

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Online Appendix

This online appendix present proofs of the propositions in the main paper.

Proof of Proposition 1

Note from the variance of the excess asset payoff that:

$$Var\left[R_{t+1} \mid \mathcal{F}_t\right] = \sigma_D^2 + \left(\frac{1}{R^f - \rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

and thus the excess volatility is driven by the $p_N^2 \sigma_N^2$ term. Consider now the expression for the less positive root of p_N from Proposition 3 in the special case in which there is an absence of government intervention:

$$p_N = \frac{1}{2\sigma_N^2} A - \sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B},$$

where:

$$\begin{aligned} A &= \frac{R^f}{\gamma}, \\ B &= \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2, \end{aligned}$$

to simplify notation. Given this expression, it follows that:

$$p_N^2 \sigma_N^2 = \frac{A^2}{2\sigma_N^2} - A\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B} - B.$$

Differentiating with respect to σ_N^2, we find with some manipulation that:

$$\frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2} = \frac{A}{2\sigma_N^2} \frac{2\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B - \frac{A}{\sigma_N^2}\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B}}{\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B}},$$

which we can factorize as:

$$\frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2} = \frac{A}{2\sigma_N^2} \frac{\left(\frac{A}{2\sigma_N^2} - \sqrt{\left(\frac{A}{2\sigma_N^2} \right)^2 - \frac{1}{\sigma_N^2} B} \right)^2}{\sqrt{\left(\frac{A}{2\sigma_N^2} \right)^2 - \frac{1}{\sigma_N^2} B}} \geq 0,$$

and, since $P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t$ with V_{t+1} and N_t independent of each other, this completes the proof. Therefore, volatility is highest close to market breakdown, when $\left(\frac{R^f}{\gamma \sigma_N^2} \right)^2 - 4 \left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 \frac{1}{\sigma_N^2} \right) = \varepsilon$ for ε arbitrarily small. Market breakdown occurs when $\varepsilon = 0$, or:

$$\sigma_N = \frac{R^f}{2\gamma \sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2}}.$$

Furthermore, as $\varepsilon \to 0$, and $\sigma_N \to \frac{R^f}{2\gamma \sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2}}$, then:

$$p_N^2 \sigma_N^2 \to \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2.$$

Consequently, the maximum conditional excess payoff variance before breakdown occurs is $Var\left[R_{t+1} \mid \mathcal{F}_t \right] \to 2 \left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 \right)$.

Proof of Proposition 2

In what follows, let τ be the maximum horizon (remaining trading periods) of all investors and $T \leq \tau$ be the horizon of an individual investor. We first solve the optimization problem of an individual investor whose current investment horizon is T. We then impose market-clearing, in which we aggregate the demands of τ vintages of investors to solve for asset prices. Finally, we consider the two cases where $\tau = 2$ and $\tau = \infty$.

We search for a covariance-stationary equilibrium, and conjecture a price process:

$$P_t = p_V V_{t+1} + p_N N_t$$

it will be convenient to define the state vector $\Psi_t = [1, v_t, N_t]$, and we search for a covariance-stationary equilibrium. Ψ_t has an AR(1) law of motion and is related to investors' returns, R_{t+1}, according to:

$$\begin{aligned} \Psi_{t+1} &= \varrho \Psi_t + \Xi \varepsilon_{t+1}, \\ R_{t+1} &= \alpha \Psi_t + \varphi' \varepsilon_{t+1}, \end{aligned}$$

where $E\left[R_{t+1}\mid\mathcal{F}_t\right]=\alpha\Psi_t$. In the limiting covariance-stationary equilibrium of the economy:

$$\begin{aligned}
\varrho &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_V & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\varphi' &= \begin{bmatrix} 1 & p_V & p_N \end{bmatrix}, \\
\Xi &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
\alpha &= \begin{bmatrix} 0 & 1+\left(\rho_V-R^f\right)p_V & -R^f p_N \end{bmatrix}.
\end{aligned}$$

As such, we can write the law of motion for an investor's wealth with horizon T at date t as:

$$W_{t+1}^{T-1}=R^f W_t^T+X_t^T\left(\alpha\Psi_t+\varphi'\varepsilon_{t+1}\right)-C_t^T.$$

Let us now conjecture that the value function of an investor with horizon T at date t, $V_t\left(T,W_t,\Psi_t\right),$ takes the functional form:

$$V_t\left(T,W_t,\Psi_t\right)=-\exp\left(-A_t^T W_t^T-\frac{1}{2}\Psi_t' B_t^T\Psi_t\right).$$

Substituting this expression into the recursive formulation of the investor's problem, we find that:

$$V_t=\sup_{\left\{C_t^T,X_t\right\}}-\exp\left(-\gamma C_t^T\right)-\beta E\left[\exp\left(\begin{array}{c} -A_{t+1}^{T-1}R^f\left(W_t^T-C_t^T\right)-A_{t+1}^{T-1}X_t^T\alpha\Psi_t-A_{t+1}^{T-1}X_t^T\varphi'\varepsilon_{t+1} \\ -\frac{1}{2}\left(\varrho\Psi_t+\Xi\varepsilon_{t+1}\right)' B_{t+1}^{T-1}\left(\varrho\Psi_t+\Xi\varepsilon_{t+1}\right)\end{array}\right)\mid\mathcal{F}_t\right].$$

By completing the square, we can rewrite the above Bellman Equation as:

$$\begin{aligned}
V_t^G &= \sup_{\left\{C_t^T,X_t^T\right\}}-\exp\left(-\gamma C_t^T\right)-\frac{1}{\sqrt{|\Omega|\left|\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right|}}\beta \\
&\times\exp\left(\begin{array}{c} -A_{t+1}^{T-1}R^f\left(W_t^T-C_t^T\right)-\frac{1}{2}\Psi_t'\varrho' B_{t+1}^{T-1}\varrho\Psi_t-A_t^{T-1}X_t^T\alpha\Psi_t \\ +\frac{1}{2}\left(A_{t+1}^{T-1}X_t^T\varphi'+\Psi_t'\varrho' B_{t+1}^{T-1}\Xi\right)\left(\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right)^{-1}\left(A_t^{T-1}X_t^T\varphi+\Xi' B_{t+1}^{T-1\prime}\varrho\Psi_t\right)\end{array}\right).
\end{aligned}$$

Taking the first-order conditions with respect to X_t^G and C_t^T, we find that the optimal position and consumption of the government take the form:

$$X_t^T=\frac{\alpha-\varphi'\left(\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right)^{-1}\Xi' B_{t+1}^{T-1\prime}\varrho}{A_{t+1}^{T-1}\varphi'\left(\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right)^{-1}\varphi}\Psi_t,$$

and:

$$\begin{aligned}
C_t^T &= \frac{A_{t+1}^{T-1}R^f}{\gamma+A_{t+1}^{T-1}R^f}W_t^T+\frac{1}{\gamma+A_{t+1}^{T-1}R^f}\log\left(\frac{\gamma}{A_{t+1}^{T-1}\beta R^f}\sqrt{|\Omega|\left|\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right|}\right) \\
&+\frac{1}{2\left(\gamma+A_{t+1}^{T-1}R^f\right)}\Psi_t'\varrho' B_{t+1}^{T-1}\varrho\Psi_t+\frac{A_{t+1}^{T-1}}{\gamma+A_{t+1}^{T-1}R^f}X_t^T\alpha\Psi_t \\
&-\frac{1}{2\left(\gamma+A_{t+1}^{T-1}R^f\right)}\left(A_{t+1}^{T-1}X_t^T\varphi'+\Psi_t'\varrho' B_{t+1}^{T-1}\Xi\right)\left(\Xi' B_{t+1}^{T-1}\Xi+\Omega^{-1}\right)^{-1}\left(A_{t+1}^{T-1}X_t^T\varphi+\Xi' B_{t+1}^{T-1\prime}\varrho\Psi_t\right).
\end{aligned}$$

Substituting for X_t^T, optimal consumption takes the form:

$$C_t^T = \frac{A_{t+1}^{T-1} R^f}{\gamma + A_{t+1}^{T-1} R^f} W_t^T + \bar{C}_{t+1}^{T-1} + \frac{1}{2\left(\gamma + A_{t+1}^{T-1} R^f\right)} \Psi_t' \Lambda_{t+1}^{T-1} \Psi_t,$$

where:

$$\begin{aligned}
\bar{C}_{t+1}^{T-1} &= \frac{1}{\gamma + A_{t+1}^{T-1} R^f} \log\left(\frac{\gamma}{\beta A_{t+1}^{T-1} R^f} \sqrt{|\Omega| \left|\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right|}\right), \\
\Lambda_{t+1}^{T-1} &= \varrho' \left(B_{t+1}^{T-1} - B_{t+1}^{T-1} \Xi \left(\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right)^{-1} \Xi' B_{t+1}^{T-1\prime}\right) \varrho \\
&\quad + \frac{\left(\alpha - \varphi' \left(\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right)^{-1} \Xi' B_{t+1}^{T-1\prime} \varrho\right)' \left(\alpha - \varphi' \left(\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right)^{-1} \Xi' B_{t+1}^{T-1\prime} \varrho\right)}{\varphi' \left(\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right)^{-1} \varphi}.
\end{aligned}$$

Substituting the optimal policies into the maximized Bellman Equation, we find that:

$$\gamma C_t^T - A_t^T W_t^T - \frac{1}{2} \Psi_t' B_t^T \Psi_t = \log\left(1 + \frac{\gamma}{A_{t+1}^{T-1} R^f}\right),$$

from which follows that:

$$\begin{aligned}
A_t^T &= \frac{\gamma}{R^f} \frac{R^f - 1}{1 - \left(R^f\right)^{t-T}}, \\
\bar{C}_{t+1}^{T-1} &= \frac{A_t^T}{\gamma A_{t+1}^{T-1} R^f} \log\left(\frac{\gamma}{\beta A_{t+1}^{T-1} R^f} \sqrt{|\Omega| \left|\Xi' B_{t+1}^{T-1} \Xi + \Omega^{-1}\right|}\right),
\end{aligned}$$

with $A_T = \gamma$.

Furthermore, B_t satisfies the finite difference system of equations:

$$\frac{1}{2} \Psi_t' \left(2\left(\gamma \bar{C}_{t+1}^{T-1} + \log\left(\frac{R^f - 1}{R^f}\right) \mathbf{e}_{5\times1}^1 \mathbf{e}_{5\times1}^{1\prime}\right) + \frac{1}{R^f} \Lambda_{t+1}^{T-1} - B_t^T\right) \Psi_t = 0,$$

and since they must hold for arbitrary Ψ_t, it follows that B_t^T is determined from B_{t+1}^{T-1} by:

$$B_t^T = \frac{1}{R^f} \Lambda_{t+1}^{T-1} + 2\left(\gamma \bar{C}_{t+1}^{T-1} + \log\left(\frac{R^f - 1}{R^f}\right)\right) \mathbf{e}_{3\times1}^1 \mathbf{e}_{3\times1}^{1\prime},$$

where $\mathbf{e}_{3\times1}^1$ is the 3×1 basis vector with first entry 1 and remaining entries 0. Recognizing that at the final date for a cohort;

$$B_t^0 = 0_{3\times3},$$

it follows that one can iterate backward in time to arrive at B_t^T. In the limit of arbitrarily large T, one can find the stationary fixed point $B_t \to B$ since all variances and covariances governing the law of motion of Ψ and the mapping to R_{t+1} are stationary.

The second-order optimality condition for the investor's problem is that, at the fixed-point, $\Xi' B_t^T \Xi + \Omega^{-1}$ is positive definite. Given the definition of Ξ, this condition reduces to:

$$\left(\Xi' B_t^T \Xi + \Omega^{-1}\right)^{-1} = \begin{bmatrix} \sigma_D^{-2} & 0 & 0 \\ 0 & B_{t,22}^T + \sigma_v^{-2} & B_{t,23}^T \\ 0 & B_{t,32}^T & B_{t,33}^T + \sigma_N^{-2} \end{bmatrix}^{-1} = \begin{bmatrix} \sigma_D^2 & 0 & 0 \\ 0 & \frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} & -\frac{B_{t,23}^T}{\Delta_t} \\ 0 & -\frac{B_{t,32}^T}{\Delta_t} & \frac{B_{t,22}^T + \sigma_V^{-2}}{\Delta_t} \end{bmatrix},$$

being positive definite, where:

$$\Delta_t = \left(B_{t,22}^T + \sigma_V^{-2}\right)\left(B_{t,33}^T + \sigma_N^{-2}\right) - B_{t,23}^T B_{t,32}^T,$$

which we will verify in the sequel that it is always satisfied.

This completes our characterization of the solution to dynamic optimization problem. The optimal consumption and investment plans can be summarized as:

$$\begin{aligned} C_t^T &= \frac{R^f - 1}{R^f} W_t^T - \frac{1}{\gamma R^f} \log\left(\left(R^f - 1\right)\beta\sqrt{|\Omega|\,|\Xi' B_t^T \Xi + \Omega^{-1}|}\right) + \frac{1}{2\gamma R^f} \Psi_t' \Lambda_{t+1}^{T-1} \Psi_t, \\ X_t^T &= R^f \frac{1 - \left(R^f\right)^{t-T}}{R^f - 1} \frac{\alpha - \varphi' \left(\Xi' B_t^T \Xi + \Omega^{-1}\right)^{-1} \Xi' B_t^{T\prime} \varrho}{\gamma \varphi' \left(\Xi' B_t^T \Xi + \Omega^{-1}\right)^{-1} \varphi} \Psi_t. \end{aligned}$$

Given the equivalence of the sequential and dynamic problems by the Dynamic Programming Principle, the solution solves the investor's problem.

Given our expression for $\left(\Xi' B_t^T \Xi + \Omega^{-1}\right)^{-1}$, it follows that:

$$X_t = \sum_{t'=t}^{T} \left(1/R^f\right)^{t'-t} \frac{1}{\gamma} \frac{\left[\begin{array}{c} \left(1 + \left(\rho_V - R^f\right) p_V\right) V_t - R^f p_N N_t - \left(\frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V - \frac{B_{t,32}^T}{\Delta_t} p_N\right)\left(B_{t,12}^T + B_{t,22}^T \rho_V V_t\right) \\ + \left(\frac{B_{t,22}^T + \sigma_V^{-2}}{\Delta_t} p_N - \frac{B_{t,23}^T}{\Delta_t} p_V\right)\left(B_{t,13}^T + B_{t,32}^T \rho_V V_t\right) \end{array}\right]}{\sigma_D^2 + \frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V^2 - \frac{B_{t,23}^T + B_{t,32}^T}{\Delta_t} p_N p_V + \frac{B_{t,22}^T + \sigma_v^{-2}}{\Delta_t} p_N^2}$$

Since there are overlapping generations of investors with horizons from 1 to τ, it follows that there is a stationary equilibrium for the asset price. Market-clearing and matching coefficients implies that

$$\begin{aligned} -\frac{1}{T}\sum_{t=0}^{T}\left(1 - \left(R^f\right)^{t-T}\right) \frac{\left(\frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_{tt}} p_V - \frac{B_{t,32}^T}{\Delta_t} p_N\right) B_{t,12}^T + \left(-\frac{B_{t,22}^T + \sigma_V^{-2}}{\Delta_t} p_N + \frac{B_{t,23}^T}{\Delta_t} p_V\right) B_{t,13}^T}{\sigma_D^2 + \frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V^2 - \frac{B_{t,23}^T + B_{t,32}^T}{\Delta_t} p_N p_V + \frac{B_{t,22}^T + \sigma_v^{-2}}{\Delta_t} p_N^2} &= 0, \\ \frac{1}{T}\sum_{t=0}^{T}\left(1 - \left(R^f\right)^{t-T}\right) \frac{\left[\begin{array}{c} 1 + \left(\rho_V - R^f\right) p_V - \left(\frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V - \frac{B_{t,32}^T}{\Delta_{tt}} p_N\right) \rho_v B_{t,22}^T \\ + \left(-\frac{B_{t,22}^T + \sigma_V^{-2}}{\Delta_t} p_N + \frac{B_{t,23}^T}{\Delta_t} p_V\right) \rho_V B_{t,23}^T \end{array}\right]}{\sigma_D^2 + \frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V^2 - \frac{B_{t,23}^T + B_{t,32}^T}{\Delta_t} p_N p_V + \frac{B_{t,22}^T + \sigma_V^{-2}}{\Delta_t} p_N^2} &= 0, \\ p_N \frac{1}{T}\sum_{t=0}^{T} \frac{1 - \left(R^f\right)^{t-T}}{\sigma_D^2 + \frac{B_{t,33}^T + \sigma_N^{-2}}{\Delta_t} p_V^2 - \frac{B_{t,23}^T + B_{32}^T}{\Delta_t} p_N p_V + \frac{B_{t,22}^T + \sigma_v^{-2}}{\Delta_t} p_N^2} &= \frac{R^f - 1}{R^f} \frac{1}{R^f} \gamma. \end{aligned}$$

Conjecture a stationary solution in which:

$$1+\left(\rho_V - R^f\right) p_V - \left(\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t} p_V - \frac{B^T_{t,32}}{\Delta_t} p_N\right) \rho_V B^T_{t,22} + \left(-\frac{B^T_{t,22}+\sigma_V^{-2}}{\Delta_t} p_N + \frac{B^T_{t,23}}{\Delta_t} p_V\right) \rho_V B^T_{t,23} = 0,$$

for each t, then it then follows that the lower right minor of B^T_t satisfies the recursion:

$$R^f \begin{bmatrix} B^T_{t-1,22} & B^T_{t-1,23} \\ B^T_{t-1,32} & B^T_{t-1,33} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{array}{c} B^T_{t,22}\left(1-\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t}B^T_{t,22}+\frac{B^T_{t,23}}{\Delta_t}B^T_{t,23}\right)\rho_v^2 \\ -\left(\frac{B^T_{t,22}+\sigma_v^{-2}}{\Delta_t}B^T_{t,23}-\frac{B^T_{t,32}}{\Delta_t}B^T_{t,22}\right)\rho_V^2 B^T_{t,23} \end{array} & 0 \\ 0 & \frac{\left(R^f p_N\right)^2}{\sigma_D^2+\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t}p_V^2-\frac{B^T_{t,23}+B^T_{t,32}}{\Delta_t}p_N p_V+\frac{B^T_{t,22}+\sigma_V^{-2}}{\Delta_t}p_N^2} \end{bmatrix},$$

from which follows that:

$$B^T_{t,23} = B^T_{t,32} = 0.$$

Consequently, the above further reduces to:

$$R^f \begin{bmatrix} B^T_{t-1,22} & 0 \\ 0 & B^T_{t-1,33} \end{bmatrix} = \begin{bmatrix} B^T_{t,22}\left(1-\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t}B^T_{t,22}\right)\rho_V^2 & 0 \\ 0 & \frac{\left(R^f p_N\right)^2}{\sigma_D^2+\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t}p_V^2+\frac{B^T_{t,22}+\sigma_V^{-2}}{\Delta_t}p_N^2} \end{bmatrix},$$

and we see that:

$$B^T_{t-1,22} = B^T_{t,22}\left(1-\frac{B^T_{t,33}+\sigma_N^{-2}}{\Delta_t}B^T_{t,22}\right)\rho_V^2 = \frac{\sigma_V^{-2}}{B^T_{t,22}+\sigma_V^{-2}}\rho_V^2 B^T_{t,22} = 0,$$

since $B^0_{t,22} = 0$, from which follows that:

$$p_V = \frac{1}{R^f - \rho_V},$$

and consequently:

$$B^T_{t-1,33} = \frac{R^f p_N^2}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2 + \frac{p_N^2}{B^T_{t,33}+\sigma_N^{-2}}}.$$

Since the recursion for B^T_t does not depend on time-varying objects, we drop the t subscript and index B^T_t only by the investor's remaining horizon, B^T_{33}. Since all coefficients on the RHS are positive, that $B^T_{t-1,33} > B^T_{t,33}\ \forall\ t$. It then follows that:

$$\left(\Xi' B^T_t \Xi + \Omega^{-1}\right)^{-1} = \begin{bmatrix} \sigma_D^2 & 0 & 0 \\ 0 & \sigma_V^2 & 0 \\ 0 & 0 & \frac{1}{B^T_{33}+\sigma_N^{-2}} \end{bmatrix},$$

which is always positive definite since $B_{33}^T \geq 0$.

In addition, the asset demand of investor of generation $t's$ asset demand is given by:

$$X_t^T = -\frac{R^f}{\gamma}\frac{1-\left(R^f\right)^{t-T}}{R^f-1}\frac{R^f p_N N_t}{\sigma_D^2+\left(\frac{\sigma_v}{R^f-\rho_v}\right)^2+\frac{p_N^2}{B_{33}^T+\sigma_N^{-2}}}.$$

Substituting this demand into market-clearing along with the definition of B_{33}^T, we arrive at:

$$\frac{1}{T}\sum_{t=0}^{T}\left(1-\left(1/R^f\right)^{T-t}\right)B_{33}^{T-1} = \frac{R^f-1}{R^f}\gamma p_N.$$

Consider now a more conservative economy in which everyone is of the investor cohort with the maximum horizon of τ. Then:

$$B_{33}^{\tau} = \frac{R^f-1}{1-\left(1/R^f\right)^T}\frac{\gamma}{R^f}p_N.$$

This implies the market-clearing condition for N_t can be expressed as:

$$\frac{p_N^2\sigma_N^2}{1+B_{33}^{\tau}\sigma_N^2} - \frac{1-\left(1/R^f\right)^T}{R^f-1}\frac{\left(R^f\right)^2}{\gamma}p_N + \sigma_D^2 + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2 = 0.$$

Consider case that $\tau = 2$, then p_N solves the quartic polynomial:

$$\begin{aligned} 0 &= p_N^4\sigma_N^4 - \frac{\left(1+R^f\right)^2}{\gamma}p_N^3\sigma_N^2 + \left(2+R^f\right)\left(\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\right)p_N^2\sigma_N^2 \\ &\quad -\left(\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\right)\frac{1+R^f}{\gamma}p_N + \left(\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\right)^2. \end{aligned}$$

By Descartes' Rule of Signs, this polynomial has zero positive real roots and either zero, two, or four positive real roots. This quartic polynomial has no real solutions for γ sufficiently large, as the limiting polynomial as $\gamma \to \infty$ is:

$$0 = p_N^4\sigma_N^4 + \left(2+R^f\right)\left(\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\right)p_N^2\sigma_N^2 + \left(\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\right)^2,$$

which has no real roots. Consequently, when investors have a two-period horizon, the market can still break down if risk aversion is sufficiently large. The issue is even more severe in the true economy, since half of the investors have a shorter one-period horizon, and less risk-bearing tolerance than the two-period investors.

Consider instead the limit that $\tau \to \infty$, and investors have an infinite horizon. Then, B_{33}^{∞} can be solved as the fixed-point of the recursion for B_{33}^{T}, and p_N solves the fixed-point problem from market-clearing:

$$\frac{R^f - 1}{\left(R^f\right)^2}\gamma\left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + \frac{p_N^2}{-\frac{R^f-1}{R^f}\gamma p_N + \sigma_N^{-2}}\right) = p_N.$$

It follows that p_N is then given explicitly by:

$$\begin{aligned} p_N &= \frac{\gamma\left(\sigma_D^2 + \frac{1}{R^f}\left(\frac{\sigma_V}{1-\rho_V}\right)^2\right) - \sigma_N^{-2}\frac{R^f}{\gamma}}{2\left(R^f - 1\right)} \\ &\quad + \sqrt{\left(\frac{\sigma_N^{-2}\frac{R^f}{\gamma} - \gamma\left(\sigma_D^2 + \frac{1}{R^f}\left(\frac{\sigma_V}{1-\rho_V}\right)^2\right)}{2\left(R^f - 1\right)}\right)^2 + \frac{\sigma_N^{-2}\left(\sigma_D^2 + \frac{1}{R^f}\left(\frac{\sigma_V}{1-\rho_V}\right)^2\right)}{R^f - 1}}, \end{aligned}$$

which always exists.

Consequently, it follows that for short horizon investors, markets can break down because investors are too risk averse, while with infinite horizon investors, a market equilibrium always exists.

Proof of Proposition 3

We derive the perfect information equilibrium with trading by the government. We first conjecture that, when V_{t+1} and N_t are observable to the government and investors, the stock price takes the linear form:

$$P_t = p_V V_{t+1} + p_N N_t + p_g G_t.$$

Given that dividends are $D_t = V_t + \sigma_D \varepsilon_t^D$, the stock price must react to a deterministic unit shift in V_{t+1} by the present value of dividends deriving from that shock, $\frac{1}{R^f - \rho_V}$, it follows that $p_V = \frac{1}{R^f - \rho_V}$. The innovations to V_{t+1} and N_t are the only source of risk and, from the perspective of all economic agents, the conditional expectation and variance of R_{t+1} are:

$$\begin{aligned} E\left[R_{t+1} \mid \mathcal{F}_t\right] &= -p_N R^f N_t - R^f p_g G_t, \\ Var\left[R_{t+1} \mid \mathcal{F}_t\right] &= \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2. \end{aligned}$$

Since all investors are identical when V_t and N_t are observable, it follows that in the CARA-Normal environment all investors have an identical mean-variance demand for the risky asset:

$$X_t^S = \frac{1}{\gamma}\frac{E\left[R_{t+1} \mid \mathcal{F}_t\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t\right]} = -\frac{1}{\gamma}\frac{p_N R^f N_t + R^f p_g G_t}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2\sigma_N^2 + p_g^2\sigma_G^2}.$$

In the government's intervention rule:

$$X_t^G = -\vartheta_N N_t + \vartheta_N \sigma_N G_t,$$

Finally, by imposing market-clearing, we arrive at:

$$\begin{aligned} N &= \frac{1}{\gamma}\frac{p_N R^f N}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2\sigma_N^2 + p_g^2\sigma_G^2} + \vartheta_N N, \\ \vartheta_N \sigma_N^2 G_t &= \frac{1}{\gamma}\frac{R^f p_g}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2\sigma_N^2 + p_g^2\sigma_G^2} G_t \end{aligned}$$

which, by matching coefficients, reveals that:

$$\begin{aligned} \frac{1}{\gamma}\frac{p_N R^f}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2\sigma_N^2 + p_g^2\sigma_G^2} + \vartheta_N &= 1, \\ p_N \frac{\vartheta_N}{1 - \vartheta_N}\sigma_N &= p_g. \end{aligned}$$

This confirms the conjectured equilibrium.

Rearranging this equation for p_N, and substituting for p_g, we arrive at the quadratic equation for p_N:

$$\left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2 \sigma_G^2\right) p_N^2 - \frac{R^f}{\gamma \sigma_N^2 (1 - \vartheta_N)} p_N + \frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 \frac{1}{\sigma_N^2} = 0, \qquad \text{(IA.1)}$$

from which follows that p_N has two roots:

$$p_N(\vartheta_N) = \frac{1}{2}\frac{\frac{R^f}{\gamma\sigma_N^2(1-\vartheta_N)} \pm \sqrt{\left(\frac{R^f}{\gamma\sigma_N^2(1-\vartheta_N)}\right)^2 - 4\left(1 + \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2\frac{1}{\sigma_N^2}\right)}}{1 + \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2}.$$

Recognizing that two positive solutions for p_N exist if the expression under the square root is nonnegative, it follows that the market breaks down occurs whenever:

$$R^f < 2(1 - \vartheta_N)\gamma\sqrt{\left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2\sigma_G^2\right)\left(\sigma_D^2\sigma_N^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2\sigma_N^2\right)}.$$

Consequently, market breakdown occurs when σ_N is sufficiently large.

Given that:

$$\begin{aligned} Var\left(\Delta P_t \mid \mathcal{F}_{t-1}\right) &= Var\left(P_{t+1} - \frac{1}{R^f - \rho_V} V_{t+1} \mid \mathcal{F}_{t-1}\right) \\ &= \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2 \\ &= \left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2 \sigma_G^2\right) p_N^2 \sigma_N^2, \end{aligned}$$

substituting for p_g, it follows that regardless of whether the government is concerned with price volatility or price informativeness, reducing the price variance from noise trading, $p_N^2 \sigma_N^2$, would accomplish both objectives since:

$$p_N^2 \sigma_N^2 = -\frac{-\frac{R^f}{\gamma(1-\vartheta_N)} p_N + \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2}{1 + \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2 \sigma_G^2},$$

is increasing in σ_N^2 through p_N.

To establish that the linear equilibrium is the unique, symmetric equilibrium, we express each investor's optimization problem as:

$$U_t = \max_{X_t} E\left[e^{-\gamma\left(R\bar{W} + X_t\left(V_{t+1} + \sigma_D \varepsilon_{t+1}^D + P_{t+1} - R P_t\right)\right)} \mid \mathcal{F}_t\right]$$

For an arbitary price function P_t, the FOC for the investor's holding of the risky asset X_t is:

$$E\left[\left(V_{t+1} + \sigma_D \varepsilon_{t+1}^D + P_{t+1} - R^f P_t\right) e^{-\gamma X_t\left(V_{t+1} + \sigma_D \varepsilon_{t+1}^D + P_{t+1} - R P_t\right)} \mid \mathcal{F}_t\right] = 0.$$

Substituting this with the market-clearing condition:

$$X_t = -\left(1 - \vartheta_N\right) N_t - \vartheta_N \sigma_N G_t,$$

we arrive at:

$$E\left[\left(V_{t+1} + \sigma_D \varepsilon_{t+1}^D + P_{t+1} - R^f P_t\right) e^{\gamma((1-\vartheta_N)N_t + \vartheta_N \sigma_N G_t,)\left(V_{t+1} + \sigma_D \varepsilon_{t+1}^D + P_{t+1} - R P_t\right)} \mid \mathcal{F}_t\right] = 0.$$

Since P_{t+1} cannot be a function of ε_{t+1}^D, as P_{t+1} is forward-looking for the new generation of investors at time $t+1$, the above can be rewritten as:

$$\begin{aligned} P_t = & \frac{1}{R^f} V_{t+1} + \frac{\gamma}{R^f} \sigma_D^2 \left(\left(1 - \vartheta_N\right) N_t + \vartheta_N \sigma_N G_t\right) \\ & + \frac{1}{R^f} E\left[P_{t+1} \frac{e^{\gamma((1-\vartheta_N)N_t + \vartheta_N \sigma_N G_t) P_{t+1}}}{E\left[e^{\gamma((1-\vartheta_N)N_t + \vartheta_N \sigma_N G_t) P_{t+1}} \mid \mathcal{F}_t\right]} \mid \mathcal{F}_t\right], \end{aligned} \tag{IA.2}$$

where we have used the properties of log-normal random variables to complete the square in the pdf and solve explicitly for the ε_{t+1}^{D} term. This defines a functional equation, whose fixed point is the price functional P_t. To see that the linear equilibrium we derived above solves this functional equation, we rewrite equation (IA.2) as:

$$P_t = \frac{1}{R^f} V_{t+1} + \frac{\gamma}{R^f} \sigma_D^2 \left(\left(1 - \vartheta_N\right) N_t + \vartheta_N \sigma_N G_t \right) + \frac{1}{R^f} \left. \left| \partial_u \log E\left[e^{u P_{t+1}} \mid \mathcal{F}_t \right] \right|_{u = -\gamma\left(\left(1-\vartheta_N\right) N_t + \vartheta_N \sigma_N G_t\right)} \right. ,$$

and conjecture that $P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t + p_g G_t$, from which follows that p_N satisfies the recursion:

$$p_{N,t} = \frac{\gamma\left(1 - \vartheta_N\right)}{R^f} \left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 + \left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N} \right)^2 \sigma_G^2 \right) \sigma_N^2 p_{N,t+1}^2 \right).$$

Suppose there is some final date $T >> 0$. On this final date, $P_T = 0$ since there is no salvage value to the asset. Then, as time goes backward, this recursion converges after a sufficiently long period of time to:

$$p_{N,t} \to_{t \to 0} \frac{1}{2} \frac{\frac{R^f}{\gamma \sigma_N^2 (1-\vartheta_N)} - \sqrt{\left(\frac{R^f}{\gamma \sigma_N^2 (1-\vartheta_N)} \right)^2 - 4 \left(1 + \left(\frac{\vartheta_N}{1-\vartheta_N} \right)^2 \sigma_G^2 \right) \left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 \frac{1}{\sigma_N^2} \right)}}{1 + \left(\frac{\vartheta_N}{1-\vartheta_N} \right)^2 \sigma_G^2},$$

which is the more stable of the two positive roots from the infinite horizon problem if:

$$R^f < 2\left(1 - \vartheta_N\right) \gamma \sqrt{\left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N} \right)^2 \sigma_G^2 \right) \left(\sigma_D^2 \sigma_N^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 \sigma_N^2 \right)},$$

and:

$$p_{N,t} \to_{t \to 0} \infty,$$

otherwise. Consequently, we can interpret market breakdown as an unstable backward recursion in which illiquidity is growing each period as volatility diverges. Interestingly, we obtain the more positive root for the fixed point for p_N from (IA.1) from the forward recursion:

$$p_{N,t+1} = \frac{\gamma\left(1 - \vartheta_N\right)}{R^f} \left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V} \right)^2 + \left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N} \right)^2 \sigma_G^2 \right) \sigma_N^2 p_{N,t}^2 \right).$$

Consequently, the more positive root is forward stable, but backward unstable.

Finally, notice from the recursion (IA.2) that, if the price at date $t+1$, P_{t+1}, is linear in $\{V_{t+1}, N_{t+1}, G_{t+1}\}$, and therefore normally distributed, then $\log E\left[e^{uP_{t+1}} \mid \mathcal{F}_t\right] =$

$uE\left[P_{t+1} \mid \mathcal{F}_t\right] + \frac{1}{2}u^2 Var\left[P_{t+1} \mid \mathcal{F}_t\right]$, or the moment-generating function for the normally distributed price. It then follows that the only solution is

$$\begin{aligned} P_t &= \frac{1}{R^f}V_{t+1} + \frac{\gamma}{R^f}\sigma_D^2\left(\left(1-\vartheta_N\right)N_t + \vartheta_N\sigma_N G_t\right) + E\left[P_{t+1} \mid \mathcal{F}_t\right] \\ &\quad + \frac{\gamma}{R^f}\left(\left(1-\vartheta_N\right)N_t + \vartheta_N\sigma_N G_t\right)Var\left[P_{t+1} \mid \mathcal{F}_t\right], \end{aligned}$$

and it follows that P_t is linear. Consequently, the linear equilibrium with the less positive p_N root of (IA.1) is the unique, backward stable equilibrium as the limit of the finite horizon problem.

Proof of Proposition A1

Based on Proposition A3 and Proposition A5, in the special case of no government intervention, the steady-state conditional means of the Kalman Filter, $\left(\hat{V}_{t+1}^M, \hat{N}_t^M\right)$, have a law of motion that satisfies:

$$\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \end{bmatrix} = \begin{bmatrix} \rho_V & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{V}_t^M \\ \hat{N}_{t-1}^M \end{bmatrix} + \mathbf{k}_t^M \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^H - p_V\rho_V\hat{V}_t^M \end{bmatrix},$$

where:

$$\mathbf{k}^M = \begin{bmatrix} \rho_V\Sigma^{M,VV} & p_V\left(\rho_V^2\Sigma^{M,VV} + \sigma_V^2\right) \\ 0 & p_N\sigma_N^2 \end{bmatrix}\left(\Omega^M\right)^{-1}$$

is the Kalman Gain, and the conditional variance Σ^M satisfies the Ricatti Equation:

$$\begin{aligned} \Sigma^M &= \begin{bmatrix} \rho_V & 0 \\ 0 & 0 \end{bmatrix}\Sigma^M\begin{bmatrix} \rho_V & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \sigma_V^2 & 0 \\ 0 & \sigma_N^2 \end{bmatrix} \\ &\quad - \mathbf{k}^M\begin{bmatrix} \rho_V\Sigma^{M,VV} & p_V\left(\rho_V\Sigma^{M,VV} + \sigma_V^2\right) \\ 0 & p_N\sigma_N^2 \end{bmatrix}', \end{aligned}$$

and that:

$$\begin{aligned} \Omega^M &= Var\left[\begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V\rho_V\hat{V}_t^M \end{bmatrix} \mid \mathcal{F}_{t-1}^M\right] \\ &= \begin{bmatrix} \Sigma^{M,VV} + \sigma_D^2 & p_V\rho_V\Sigma^{M,VV} \\ p_V\rho_V\Sigma^{M,VV} & p_V^2\left(\rho_V^2\Sigma^{M,VV} + \sigma_V^2\right) + p_N^2\sigma_N^2 \end{bmatrix}, \end{aligned}$$

From Proposition A4, we further recognize that:

$$\begin{aligned} \Sigma^{M,VN} &= -\frac{p_V}{p_N}\Sigma^{M,VV}, \\ \Sigma^{M,NN} &= \left(\frac{p_V}{p_N}\right)^2\Sigma^{M,VV}. \end{aligned}$$

Consequently, recognizing that all four implied coefficient equations for Σ^M are degenerate, the equation that identifies $\Sigma^{M,VV}$ reduces to:

$$\frac{\Sigma^{M,VV}}{\sigma_V^2} = \frac{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2\left(\left(1+\rho_V^2\left(\frac{\sigma_D}{\sigma_V}\right)^2\right)\frac{\Sigma^{M,VV}}{\sigma_V^2}+\left(\frac{\sigma_D}{\sigma_V}\right)^2\right)}{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2\left(\frac{\Sigma^{M,VV}}{\sigma_V^2}+\left(\frac{\sigma_D}{\sigma_V}\right)^2\right)+\left(1+\rho_V^2\left(\frac{\sigma_D}{\sigma_V}\right)^2\right)\frac{\Sigma^{M,VV}}{\sigma_V^2}+\left(\frac{\sigma_D}{\sigma_V}\right)^2}.$$

From Proposition A4, investors update their beliefs from the market beliefs by Bayes' Law in accordance with a linear updating rule. The posterior of investor i is $N\left(\hat{V}_{t+1}^i, \Sigma_t^s(i)\right)$, where $\hat{V}_{t+1}^i = E\left[V_{t+1} \mid \mathcal{F}_t^i\right]$ and $\Sigma^s(i) = E\left[\left(V_{t+1}-\hat{V}_{t+1}^i\right)^2 \mid \mathcal{F}_t^i\right]$ are given by:

$$\hat{V}_{t+1}^i = \hat{V}_{t+1}^M + \frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}\left(s_t^i - \hat{V}_{t+1}^M\right),$$

and:

$$\Sigma^s(i)^{-1} = \left(\Sigma^{M,VV}\right)^{-1} + \tau_s.$$

This characterizes the beliefs of investors given the market beliefs.

Since the government does not trade in this benchmark, investors have no incentive to learn about the government's behavior, and therefore the information acquisition decision is trivial. Given that investors each acquire a private signal s_t^i, standard results for CARA utility with normally distributed prices and payoffs establish that the optimal trading policy of investor i, X_t^i, is given by:

$$\begin{aligned} X_t^i &= \frac{E\left[D_{t+1}+P_{t+1}-R^fP_t \mid \mathcal{F}_t^i\right]}{\gamma Var\left[D_{t+1}+P_{t+1} \mid \mathcal{F}_t^i\right]} \\ &= \frac{\left(\begin{array}{c}\left(1+p_V\left(\rho_V-R^f\right)\right)\left(\hat{V}_{t+1}^i-\hat{V}_{t+1}^M\right)-p_NR^f\hat{N}_t^i \\ +\begin{bmatrix} p_{\hat{V}}-p_V \\ 0\end{bmatrix}'\mathbf{k}^M\begin{bmatrix}\hat{V}_{t+1}^i-\hat{V}_{t+1}^M \\ p_V\rho_V\left(\hat{V}_{t+1}^i-\hat{V}_{t+1}^M\right)\end{bmatrix}\end{array}\right)}{\gamma\varphi'\Omega(i)\varphi}, \end{aligned}$$

where:

$$\varphi = \begin{bmatrix}1\\1\end{bmatrix} + \mathbf{k}^{M\prime}\begin{bmatrix}p_{\hat{V}}-p_V\\0\end{bmatrix},$$

and:

$$\Omega(i) = \Omega^M - \begin{bmatrix}1\\p_V\rho_V\end{bmatrix}\frac{\left(\Sigma^{M,VV}\right)^2}{\Sigma^{M,VV}+\tau_s^{-1}}\begin{bmatrix}1\\p_V\rho_V\end{bmatrix}',$$

is the conditional variance of D_{t+1} and P_{t+1} with respect to $\mathcal{F}_t^i$. we can rewrite the above as:

$$X_t^i = \frac{\left(1+p_V\left(\rho_V-R^f\right)+\begin{bmatrix}p_{\hat{V}}-p_V\\0\end{bmatrix}'\mathbf{k}^M\begin{bmatrix}1\\p_V\rho_V\end{bmatrix}\right)\left(\hat{V}_{t+1}^i-\hat{V}_{t+1}^M\right)-p_NR^f\hat{N}_t^i}{\gamma\varphi'\Omega(i)\varphi}.$$

Substituting for $\hat{V}_{t+1}^{i}$, and recognizing from above that:

$$\hat{N}_t^M = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}_{t+1}^M\right),$$

and therefore that:

$$\hat{N}_t^i = \hat{N}_t^M - \frac{p_V}{p_N}\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right) = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}_{t+1}^M\right) - \frac{p_V}{p_N}\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right),$$

we arrive at:

$$X_t^i = \frac{\left(\varphi'\begin{bmatrix} 1 \\ p_V\rho_V \end{bmatrix}\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}\left(s_t^i - \hat{V}_{t+1}^M\right) - R^f p_N N_t - R^f p_V\left(V_{t+1} - \hat{V}_{t+1}^M\right)\right)}{\gamma\varphi'\Omega\left(i\right)\varphi}.$$

Aggregating over the demand of investors and imposing market-clearing, we arrive at the two equations for p_V and p_N:

$$\begin{aligned}
\varphi'\begin{bmatrix} 1 \\ p_V\rho_V \end{bmatrix}\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}} - R^f p_V &= 0, \\
\frac{R^f p_N}{\gamma\varphi'\Omega\left(i\right)\varphi} &= 1.
\end{aligned}$$

This completes our characterization of the linear equilibrium.

We now recognize from the market-clearing condition for p_V that we can express p_V as

$$p_V = \left(\frac{1}{R^f - \rho_V} + \frac{p_{\hat{V}} - p_V}{R^f}\frac{\rho_V^2\left(1-\rho_V\right)\frac{\Sigma^{M,VV}}{\sigma_V^2}\left(\frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2\right) - \left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2\left(\frac{\sigma_D}{\sigma_V}\right)^2}{\rho_V^2\frac{\Sigma^{M,VV}}{\sigma_V^2}\left(\frac{\sigma_D}{\sigma_V}\right)^2 + \left(1 + \left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2\right)\left(\frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2\right)}\right)\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}.$$

From our implicit equation for $\Sigma^{M,VV}$ above, we can verify that the second term in parentheses is negative (assuming $p_{\hat{V}} > p_V$), from which follows that

$$p_V \leq \frac{1}{R^f - \rho_V}\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}} \leq \frac{1}{R^f - \rho_V} = p_{\hat{V}},$$

confirming the assumption.

Finally, recognizing that return volatility from the market perspective satisfies

$$\begin{aligned}
\varphi'\Omega^M\varphi &= \left(1 + 2p_{\hat{V}}\rho_V + p_V^2\rho_V^2 + 2\left(p_{\hat{V}} - p_V\right)p_V\rho_V - \left(p_{\hat{V}} - p_V\right)^2\left(1-\rho_V^2\right)\right)\Sigma^{M,VV} \\
&\quad + \sigma_D^2 + p_N^2\sigma_N^2 + p_{\hat{V}}^2\sigma_V^2
\end{aligned}$$

which makes use of the relation by the Law of Total Variance

$$\mathbf{k}^M\Omega^M\mathbf{k}^M = \begin{bmatrix} \rho_V^2\Sigma^{M,VV} + \sigma_V^2 & 0 \\ 0 & \sigma_N^2 \end{bmatrix} - \Sigma^M,$$

we can rewrite the market-clearing condition for p_N, substituting with that of p_V, as

$$\begin{aligned}\frac{R^f p_N}{\gamma} &= \left(1 + 2\left(p_{\hat{V}} + p_V p_{\hat{V}} - p_V^2\right)\rho_V + p_V^2 \rho_V^2 - \left(p_{\hat{V}} - p_V\right)^2 - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2\right) \Sigma^{M,VV} \\ &+\sigma_D^2 + p_N^2 \sigma_N^2 + p_{\hat{V}}^2 \sigma_V^2.\end{aligned}$$

Notice in the special case that $\rho_V = 0$ that

$$p_V = \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} \frac{1}{R^f},$$

and the above condition for p_N reduces to

$$\frac{R^f p_N}{\gamma} = \left(1 - \left(\frac{1}{R^f}\right)^2 \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} + \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

since $p_{\hat{V}} = \frac{1}{R^f}$. Comparing this condition for p_N to the perfect information case

$$\frac{1}{\gamma} p_N R^f = \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

we recognize since $\frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} > 0$ that the additional term from uncertainty exacerbates the market breakdown problem. To see this, we fix $\Sigma^{M,VV}$ and express the solution to p_N as

$$p_N = \frac{1}{2\sigma_N^2} A - \sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B},$$

where:

$$\begin{aligned} A &= \frac{R^f}{\gamma}, \\ B &= \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + \left(1 - \left(\frac{1}{R^f}\right)^2 \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}. \end{aligned}$$

Since $\left(1 - \frac{1}{R^2} \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} > 0$, regardless of the equilibrium value of p_N, it follows nonexistence, which occurs when

$$\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B < 0,$$

must now occur at a positive value of p_N, and that p_N is higher in the presence of informational frictions when a solution exists (by shrinking the $\sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B}$ term in the expression for p_N). From the condition for existence in Proposition 1, it follows that market-breakdown must occur at a lower value of σ_N, $\sigma_N^{**} \geq \sigma_N^*$.

Consider the other extreme of $\rho_V = 1$, then the coefficient on $\Sigma^{M,VV}$ in the expression for p_N reduces to

$$1 + 2p_{\hat{V}} + (2p_{\hat{V}} - p_V)\, p_V - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2.$$

Since $p_V \leq \frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}} p_{\hat{V}}$ and with $p_{\hat{V}} = \frac{1}{R^f - 1}$, it follows that

$$\begin{aligned}
& 1 + 2p_{\hat{V}} + (2p_{\hat{V}} - p_V)\, p_V - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2 \\
> \; & 1 + 2p_{\hat{V}} + (p_{\hat{V}} - p_V)\, p_V - p_V \left(1 + R^f\right) \\
= \; & 1 - p_V \left(R^f - 1\right) + (p_{\hat{V}} - p_V)(2 + p_V) \\
> \; & \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} + (p_{\hat{V}} - p_V)(2 + p_V) \\
> \; & 0,
\end{aligned}$$

and consequently similar arguments establish that market breakdown occurs sooner, and p_N is more positive with informational frictions. The intermediate cases ($\rho_V \in (0,1)$) follow by similar arguments.

Proof of Proposition A2

Consider now conditional price volatility

$$\begin{aligned}
Var\left[\Delta P_{t+1} \mid \; \mathcal{F}_t^M\right] &= \\
\left(\varphi - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)' \Omega^M \left(\varphi - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) &= \left(p_V^2 \rho_V^2 + 2\,(p_{\hat{V}} - p_V)\, p_V \rho_V - (p_{\hat{V}} - p_V)^2 \left(1 - \rho_V^2\right)\right) \Sigma^{M,VV} \\
& \quad + p_{\hat{V}}^2 \sigma_V^2 + p_N^2 \sigma_N^2.
\end{aligned}$$

In the special case that $\rho_V = 0$ that

$$\begin{aligned}
Var\left[\Delta P_{t+1} \mid \; \mathcal{F}_t^M\right] &= p_{\hat{V}}^2 \sigma_V^2 + p_N^2 \sigma_N^2 - (p_{\hat{V}} - p_V)^2 \, \Sigma^{M,VV} \\
&= p_{\hat{V}} \sigma_V^2 + p_N^2 \sigma_N^2 - \left(\frac{1}{R^f}\right)^2 \left(\frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)^2 \Sigma^{M,VV}.
\end{aligned}$$

From the market-clearing condition for p_N, one has that

$$p_N^2 \sigma_N^2 - \left(\frac{1}{R^f}\right)^2 \left(\frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)^2 \Sigma^{M,VV} = \frac{R^f p_N}{\gamma} - \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2 \sigma_V^2,$$

the above can be rewritten as

$$Var\left[\Delta P_{t+1} \mid \; \mathcal{F}_t^M\right] = p_{\hat{V}} \sigma_V^2 + \frac{R^f p_N}{\gamma} - \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2 \sigma_V^2.$$

Subtracting price volatility under perfect information, $p_{\hat{V}}\sigma_V^2 + \tilde{p}_N^2\sigma_N^2$, where $\tilde{p}_N$ is the coefficient on N_t under perfect information, and recognizing that

$$\tilde{p}_N^2\sigma_N^2 = \frac{1}{\gamma}\tilde{p}_N R^f - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2 \sigma_V^2,$$

from its market-clearing condition, we arrive at

$$\begin{aligned}
& Var\left[\Delta P_{t+1} \mid \mathcal{F}_t^M\right] - p_{\hat{V}}\sigma_V^2 - \tilde{p}_N^2\sigma_N^2 \\
= \; & \frac{R^f}{\gamma}\left(p_N - \tilde{p}_N\right) - \frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} \\
= \; & -\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} + p_{\hat{V}}^2\sigma_V^2 + \tilde{p}_N^2\sigma_N^2 + \frac{R^f}{\gamma\sigma_N}\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2} \\
& -\frac{R^f}{\gamma\sigma_N}\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2 - \left(1 - \left(\frac{1}{R^f}\right)^2 \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}}
\end{aligned}$$

It then follows from a first-order Taylor expansion of $-\sqrt{x-a}$ around $a = 0$, (which omits a positive residual when a is less than 1)

$$\begin{aligned}
& -\sigma_V\sqrt{\frac{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2}{\sigma_V^2} - \left(1 - \left(\frac{1}{R^f}\right)^2\frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)\frac{\frac{\Sigma^{M,VV}}{\sigma_V^2}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}} \\
= \; & -\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2} + \frac{\left(1 - \left(\frac{1}{R^f}\right)^2\frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}}{2\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2}} \\
& +O\left(\left(1 - \left(\frac{1}{R^f}\right)^2\frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)^2\left(\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)^2\right),
\end{aligned}$$

and substituting it into our expression for excess volatility that

$$\begin{aligned}
& \frac{\left(\varphi - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)'\Omega^M\left(\varphi - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) - p_{\hat{V}}\sigma_V^2 - \tilde{p}_N^2\sigma_N^2}{\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}} \\
> \; & \frac{\frac{1}{2}\frac{R^f}{\gamma\sigma_N}\left(1 - \left(\frac{1}{R^f}\right)^2\frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)}{\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{\sigma_V}{R^f}\right)^2}} - 1 \\
> \; & \frac{\frac{1}{2}\frac{R^f}{\gamma\sigma_N}\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}}{\sqrt{\frac{1}{4}\left(\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{\sigma_V}{R^f}\right)^2}} - 1,
\end{aligned}$$

from which follows that it is sufficient for the difference to be positive that

$$\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}} > \frac{\sqrt{\left(\frac{1}{2}\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{\sigma_V}{R^f}\right)^2}}{\frac{1}{2}\frac{R^f}{\gamma\sigma_N}},$$

where $\frac{\sqrt{\left(\frac{1}{2}\frac{R^f}{\gamma\sigma_N}\right)^2 - \sigma_D^2 - \left(\frac{1}{R^f}\right)^2\sigma_V^2}}{\frac{1}{2}\frac{R^f}{\gamma\sigma_N}} < 1$ when the square root is real. Notice that the RHS is decreasing in $\gamma\sigma_N$, σ_D^2, and σ_V^2, which relaxes the sufficient condition, and increasing in R^f.

Now we recognize that $\Sigma^{M,VV}$ is increasing in σ_N^2. To see this, we simplify the identifying condition for $\Sigma^{M,VV}$ to

$$\frac{\Sigma^{M,VV}}{\sigma_V^2} = \frac{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2}{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2+1},$$

and see that $\Sigma^{M,VV}$ is increasing in the noise-to-signal ratio, $\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2$. Recognizing that

$$\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}} = \frac{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2}{\left(1+\frac{\tau_s^{-1}}{\sigma_V^2}\right)\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2+\frac{\tau_s^{-1}}{\sigma_V^2}}$$

then, given the definition of p_N, we can express $\frac{p_N\sigma_N}{p_V\sigma_V}$ as

$$\begin{aligned}&\frac{p_N\sigma_N}{p_V\sigma_V}\left(\frac{\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2}{\left(1+\frac{\tau_s^{-1}}{\sigma_V^2}\right)\left(\frac{p_N\sigma_N}{p_V\sigma_V}\right)^2+\frac{\tau_s^{-1}}{\sigma_V^2}}\right)\\ = \;& -\sqrt{\left(\frac{1}{2\sigma_N}\frac{R^f}{\gamma}\right)^2 - \sigma_D^2 - \left(\frac{\sigma_V}{R^f}\right)^2 - \left(1-\left(\frac{1}{R^f}\right)^2\left(1-\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)\right)\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}}\\ &+\frac{1}{2\sigma_N}\frac{R^f}{\gamma}.\end{aligned}$$

Since the LHS is an increasing function of $\frac{p_N\sigma_N}{p_V\sigma_V}$ (since $\frac{p_N}{p_V}>0$), while the RHS is increasing in σ_N by the comparative static for p_N, it follows that $\frac{p_N\sigma_N}{p_V\sigma_V}$ is increasing in σ_N.

Since the LHS of the sufficient condition is increasing in σ_N through $\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}$, while the RHS is decreasing in σ_N, it follows that there is a critical σ_N^* such that price volatility with imperfect information is higher.

Similarly, one can express the deviation of the price from its fundamental value as

$$\begin{aligned}
& Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \ \mathcal{F}_t^M\right] \\
= \ & Var\left[\Delta P_{t+1} \mid \ \mathcal{F}_t^M\right] + \left(p_{\hat{V}}^2 - 2 p_{\hat{V}} p_V\right) \sigma_V^2 - 2 p_{\hat{V}} \left(p_{\hat{V}} - p_V\right) \left(1 - \frac{\left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2}{1 + \left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2}\right) \sigma_V^2 \\
= \ & Var\left[\Delta P_{t+1} \mid \ \mathcal{F}_t^M\right] - p_{\hat{V}}^2 \sigma_V^2 + 2 p_{\hat{V}} \left(p_{\hat{V}} - p_V\right) \Sigma^{M,VV},
\end{aligned}$$

from our definition of $\Sigma^{M,VV}$. Subtracting its perfect information counterpart, $p_N^2 \sigma_N^2$, we arrive at

$$\begin{aligned}
& Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \ \mathcal{F}_t^M\right] - p_N^2 \sigma_N^2 \\
= \ & Var\left[\Delta P_{t+1} \mid \ \mathcal{F}_t^M\right] - p_{\hat{V}}^2 \sigma_V^2 - p_N^2 \sigma_N^2 + 2 p_{\hat{V}}^2 \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}.
\end{aligned}$$

Notice when $Var\left[\Delta P_{t+1} \mid \ \mathcal{F}_t^M\right] > p_{\hat{V}}^2 \sigma_V^2 + p_N^2 \sigma_N^2$, or price volatility is higher than with perfect information, then it also follows that

$$Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \ \mathcal{F}_t^M\right] > p_N^2 \sigma_N^2.$$

Proof of Proposition A3

To arrive at the beliefs of investors and the government, we first characterize the market beliefs based on the public information set $\mathcal{F}_t^M$. To derive the market beliefs, we proceed in several steps. First, we assume the market posterior belief of (V_{t+1}, N_t, G_{t+1}) is jointly Gaussian, $(V_{t+1}, N_t, G_{t+1}) \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right), \Sigma_t^M\right)$, where:

$$\begin{aligned}
\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \\ G_t \end{bmatrix} &= E\left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{bmatrix} \mid \mathcal{F}_t^M\right], \\
\Sigma_t^M &= \begin{bmatrix} \Sigma_t^{M,VV} & \Sigma_t^{M,VN} & \Sigma_t^{M,VG_1} & 0 \\ \Sigma_t^{M,VN} & \Sigma_t^{M,NN} & \Sigma_t^{M,NG_1} & 0 \\ \Sigma_t^{M,VG_1} & \Sigma_t^{M,NG_1} & \Sigma_t^{M,G_1G_1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

Standard results for the Kalman Filter then establish that the law of motion of the conditional expectation of the market's posterior beliefs $\left(\hat{V}_{t+1}^M, \hat{N}_t^M\right)$ is:

$$\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \\ G_t \end{bmatrix} = \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_t^M \\ \hat{N}_{t-1}^M \\ \hat{G}_{t|t-1}^M \\ G_{t-1} \end{bmatrix} + \mathbf{K}_t^M \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1} \end{bmatrix},$$

where:

$$\begin{aligned} \mathbf{K}_t^M &= CoV\left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{bmatrix}, \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_{t-1}^M \\ G_t - G_{t|t-1} \end{bmatrix} \mid \mathcal{F}_{t-1}^M\right] \\ &\quad \times Var\left[\begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1} \end{bmatrix} \mid \mathcal{F}_{t-1}^M\right]^{-1}, \end{aligned}$$

is the Kalman Gain, and that the conditional variance Σ_t^M evolves deterministically according to:

$$\begin{aligned} \Sigma_t^M &= \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Sigma_{t-1}^M \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} \sigma_V^2 & 0 & 0 & 0 \\ 0 & \sigma_N^2 & 0 & 0 \\ 0 & 0 & \sigma_G^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &\quad - \mathbf{K}_t^M CoV\left[\begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1} \end{bmatrix}, \begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{bmatrix} \mid \mathcal{F}_{t-1}^M\right]. \end{aligned}$$

It is straightforward to compute that:

$$\begin{aligned} & CoV\left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{bmatrix}, \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1} \end{bmatrix} \mid \mathcal{F}_{t-1}^M\right] \\ &= \begin{bmatrix} \rho_V \Sigma_{t-1}^{M,VV} & p_V \left(\rho_V^2 \Sigma_{t-1}^{M,VV} + \sigma_V^2\right) & \rho_V \Sigma_{t-1}^{M,VG_1} \\ 0 & p_N \sigma_N^2 & 0 \\ 0 & p_G \sigma_G^2 & 0 \\ \Sigma_{t-1}^{M,VG_1} & p_V \rho_V \Sigma_{t-1}^{M,VG_1} & \Sigma_{t-1}^{M,G_1G_1} \end{bmatrix}, \end{aligned}$$

and that:

$$\begin{aligned}
\Omega_{t-1}^{M} &= Var\left[\left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V\rho_V\hat{V}_t^M \\ G_t - G_{t|t-1} \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] \\
&= \left[\begin{array}{ccc} \Sigma_{t-1}^{M,VV} + \sigma_D^2 & p_V\rho_V\Sigma_{t-1}^{M,VV} & \Sigma_{t-1}^{M,VG_1} \\ p_V\rho_V\Sigma_{t-1}^{M,VV} & p_V^2\left(\rho_V^2\Sigma_{t-1}^{M,VV} + \sigma_V^2\right) + p_N^2\sigma_N^2 + p_G^2\sigma_G^2 & p_V\rho_V\Sigma_{t-1}^{M,VG_1} \\ \Sigma_{t-1}^{M,VG_1} & p_V\rho_V\Sigma_{t-1}^{M,VG_1} & \Sigma_{t-1}^{M,G_1G_1} \end{array}\right].
\end{aligned}$$

Since $\eta_t^M \in \mathcal{F}_t^M \subseteq \mathcal{F}_t$, I can express η_t^M as:

$$\eta_t^M = p_V V_{t+1} + p_N N_t + p_G G_{t+1} = p_V\hat{V}_{t+1}^M + p_N\hat{N}_t^M + p_G\hat{G}_{t+1}^M,$$

from which follows that:

$$p_V\left(V_{t+1} - \hat{V}_{t+1}^M\right) + p_N\left(N_t - \hat{N}_t^M\right) + p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right) = 0.$$

As a consequence, it must be that the market beliefs about V_t and N_t are ex-post correlated after observing the stock price innovation process η_t^M, such that we have the three identities by taking its variance and its covariance with $V_{t+1} - \hat{V}_{t+1}^M$ and $N_t - \hat{N}_t^M$:

$$\begin{aligned}
\Sigma_t^{M,VN} &= -\frac{p_V}{p_N}\Sigma_t^{M,VV} - \frac{p_G}{p_N}\Sigma_t^{M,VG_1}, \\
\Sigma_t^{M,NN} &= -\frac{p_V}{p_N}\Sigma_t^{M,VN} - \frac{p_G}{p_N}\Sigma_t^{M,NG_1}, \\
\Sigma_t^{M,NG_1} &= -\frac{p_V}{p_N}\Sigma_t^{M,VG_1} - \frac{p_G}{p_N}\Sigma_t^{M,G_1G_1}.
\end{aligned}$$

This completes our characterization of the market's beliefs.

Proof of Proposition A4

Updating the market beliefs to each investor's private beliefs can be done in a manner similar to that in He and Wang (1995). Note that the market beliefs act as the prior for investor i who observes the normally distributed private signal s_t^i. The posterior of investor i is $N\left(\left(\hat{V}_{t+1}^i, \hat{N}_t^i, \hat{G}_{t+1|t}^i\right), \Sigma_t^i\right)$, where $\left(\hat{V}_{t+1}^i, \hat{N}_t^i, \hat{G}_{t+1|t}^i\right) = E\left[(V_t, N_t, G_{t+1}) \mid \mathcal{F}_t^i\right]$ and $\Sigma_t^s(i) = E\left[\left[\begin{array}{c} V_{t+1} - \hat{V}_{t+1}^i \\ N_t - \hat{N}_t^i \\ G_{t+1} - \hat{G}_{t+1|t}^i \end{array}\right]\left[\begin{array}{c} V_{t+1} - \hat{V}_{t+1}^i \\ N_t - \hat{N}_t^i \\ G_{t+1} - \hat{G}_{t+1|t}^i \end{array}\right]' \mid \mathcal{F}_t^i\right]$ are given by:

$$\left[\begin{array}{c} \hat{V}_{t+1}^i \\ \hat{N}_t^i \\ \hat{G}_{t+1}^i \end{array}\right] = \left[\begin{array}{c} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \end{array}\right] + \Gamma_t'\left[\begin{array}{c} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{array}\right],$$

where:

$$
\begin{aligned}
\Gamma_t' &= CoV\left[\left[\begin{array}{c} V_{t+1} \\ N_t \\ G_{t+1} \end{array}\right], \left[\begin{array}{c} s_t - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] Var\left[\left[\begin{array}{c} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{array}\right] \mid \mathcal{F}_t^M\right]^{-1} \\
&= \left[\begin{array}{cc} \Sigma_t^{M,VV} & \Sigma_t^{M,VG_1} \\ \Sigma_t^{M,VN} & \Sigma_t^{M,NG_1} \\ \Sigma_t^{M,VG_1} & \Sigma_t^{M,G_1G_1} \end{array}\right] \left[\begin{array}{cc} \Sigma_t^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma_t^{M,VG_1} \\ \Sigma_t^{M,VG_1} & \Sigma_t^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{array}\right]^{-1},
\end{aligned}
$$

and:

$$
\Sigma_t^s(i) = \Sigma_t^M - \Gamma_t^{i\prime} \left[\begin{array}{cc} \Sigma_t^{M,VV} & \Sigma_t^{M,VG_1} \\ \Sigma_t^{M,VN} & \Sigma_t^{M,NG_1} \\ \Sigma_t^{M,VG_1} & \Sigma_t^{M,G_1G_1} \end{array}\right]'.
$$

Since G_t is publicly revealed, it is common knowledge and speculators need not update their beliefs about it with their private information. This characterizes the beliefs of investors given the market's beliefs.

Proof of Proposition A5

After the system has run for a sufficiently long time, initial conditions will diminish and the conditional variance of the Kalman Filter for the market beliefs Σ_t^M will settle down to its deterministic, covariance-stationary steady-state. To see this, let us conjecture that $\Sigma_t^M \to \Sigma^M$. In this proposed steady-state, $\Gamma_t \to \Gamma$, where Γ is given by:

$$
\Gamma = \left[\begin{array}{cc} \Sigma^{M,VV} & \Sigma^{M,VG_1} \\ \Sigma^{M,VN} & \Sigma^{M,NG_1} \\ \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} \end{array}\right] \left[\begin{array}{cc} \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma^{M,VG_1} \\ \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{array}\right]^{-1}.
$$

Consequently, since Γ is indeed constant, so is $\Sigma^{M,VV}$. Furthermore, the steady-state Kalman Gain $\mathbf{K}^M$ is given by:

$$
\mathbf{K}^M = \left[\begin{array}{ccc} \rho_V \Sigma^{M,VV} & p_V\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) & \rho_V \Sigma^{M,VG_1} \\ 0 & p_N \sigma_N^2 & 0 \\ 0 & p_G \sigma_G^2 & 0 \\ \Sigma^{M,VG_1} & p_V \rho_V \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} \end{array}\right] \Omega^{M-1},
$$

where:

$$
\Omega^M = \left[\begin{array}{ccc} \Sigma^{M,VV} + \sigma_D^2 & p_V \rho_V \Sigma^{M,VV} & \Sigma^{M,VG_1} \\ p_V \rho_V \Sigma^{M,VV} & p_V^2\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2 & p_V \rho_V \Sigma^{M,VG_1} \\ \Sigma^{M,VG_1} & p_V \rho_V \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} \end{array}\right].
$$

Consequently, since we have constructed a steady-state for the Kalman Filter for the market beliefs, such a steady-state exists.

Proof of Proposition A6

Similar to the problem for the government, it is convenient to define the state vector $\Psi_t = \left[\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M, G_t\right]$ with law of motion:

$$\Psi_{t+1} = \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Psi_t + \mathbf{K}^M \varepsilon_{t+1}^M,$$

and $\varepsilon_{t+1}^M \mid \mathcal{F}_t^M \sim N\left(\mathbf{0}_{3\times 1}, \Omega^M\right)$ is given by:

$$\varepsilon_{t+1}^M = \begin{bmatrix} D_{t+1} - \hat{V}_{t+1}^M \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^M \\ G_{t+1} - \hat{G}_{t+1}^M \end{bmatrix},$$

with Ω^M given in the proof of Corollary 1.

Given that excess payoffs are normally distributed, we can decompose R_{t+1} as:

$$\begin{aligned} R_{t+1} &= E\left[R_{t+1} \mid \mathcal{F}_t^i\right] + \phi' \varepsilon_{t+1}^{S,i} \\ &= \varsigma \Psi_t + \phi' \omega \begin{bmatrix} \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma^{M,VG_1} \\ \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} + \phi' \varepsilon_{t+1}^{S,i} \\ &= \varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2} + \phi' \varepsilon_{t+1}^{S,i}, \end{aligned}$$

where:

$$\varepsilon_{t+1}^{S,i} = \begin{bmatrix} D_{t+1} - \hat{V}_{t+1}^i \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^i \\ G_{t+1} - \hat{G}_{t+1}^i \end{bmatrix},$$

and:

$$\begin{aligned} \varsigma &= \begin{bmatrix} 1 + p_{\hat{V}}\left(\rho_V - R^f\right) & -p_N R^f & p_g - R^f p_{\hat{G}} & -R^f p_g \end{bmatrix}, \\ \phi &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \mathbf{K}^{M\prime} \begin{bmatrix} p_{\hat{V}} - p_V \\ 0 \\ p_{\hat{G}} - p_G \\ p_g \end{bmatrix}. \end{aligned}$$

In this decomposition, we have updated the investor's beliefs sequentially from the market beliefs following Bayes' Rule as:

$$\begin{aligned}
& E\left[R_{t+1} \mid \mathcal{F}_t^i\right] \\
= \; & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \phi'\omega \begin{bmatrix} \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} & \Sigma^{M,VG_1} \\ \Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} \\
= \; & \varsigma\Psi_t + \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2},
\end{aligned}$$

where, as in Proposition 4:

$$\begin{aligned}
\omega &= CoV\left[\varepsilon_{t+1}^M, \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}' \mid \mathcal{F}_t^M\right] \\
&= \begin{bmatrix} \Sigma^{M,VV} & \Sigma^{M,VG_1} \\ p_V\rho_V\Sigma^{M,VV} & p_V\rho_V\Sigma^{M,VG_1} \\ \rho_V\Sigma^{M,VG_1} & \Sigma^{M,G_1G_1} \end{bmatrix}.
\end{aligned}$$

Similarly, by Bayes' Rule, $\varepsilon_{t+1}^S \mid \mathcal{F}_t^i \sim \mathcal{N}\left(\mathbf{0}_{2\times 1}, \Omega^S\right)$, where:

$$\Omega^S = \Omega^M - \frac{\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \omega'}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2}.$$

Standard results establish that the investor's problem is equivalent to the mean-variance optimization program:

$$\sup_{X_t(i)} \left\{R^f\bar{W} + X_t^i E\left[R_{t+1} \mid \mathcal{F}_t^i\right] - \frac{\gamma}{2}X_t^{i2} Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]\right\}.$$

Importantly, since the investors have to form conditional expectations about excess payoffs at $t+1$, they must form conditional expectations about the government's future trading $E\left[G_{t+1} \mid \mathcal{F}_t^i\right]$. Given that the investors are price-takers, from the FOC we see that the optimal investment of investor i in the risky asset is given by:

$$\begin{aligned}
X_t^i &= \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\
&= \frac{1}{\gamma} \frac{\varsigma\Psi_t + \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2}}{\phi'\Omega^M\phi - \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \omega'\phi}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2}}.
\end{aligned}$$

This completes our characterization of the optimal trading policy of the investors.

Proof of Proposition A7

Each investor faces the optimization problem (A1) given in the main paper. It then follows that investor i will choose to learn about the payoff fundamental V_t (i.e, $a_t^i = 1$) with probability λ:

$$\lambda = \begin{cases} 1, & Q < 0 \\ (0,1), & Q = 0 \\ 0, & Q > 0, \end{cases},$$

where:

$$Q = \phi' \left(M(0) - M(1) \right) \phi = \phi' \omega \begin{bmatrix} -\frac{1}{\Sigma^{M,VV} + \tau_s^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} \end{bmatrix} \omega' \phi.$$

Given ω, we can expand out this condition to arrive at:

$$Q = \frac{\begin{pmatrix} \left(1 + (p_{\hat{V}} - p_V) \mathbf{K}^M_{1,1} + (p_{\hat{g}} - p_g) \mathbf{K}^M_{3,1} + (p_{\hat{G}} - p_G) \mathbf{K}^M_{4,1}\right) \Sigma^{M,VG_1} \\ + \left(1 + (p_{\hat{V}} - p_V) \mathbf{K}^M_{1,2} + (p_{\hat{g}} - p_g) \mathbf{K}^M_{3,2} + (p_{\hat{G}} - p_G) \mathbf{K}^M_{4,2}\right) \\ \times \left(p_V \rho_V \Sigma^{M,VG_1} + p_g \Sigma^{M,G_1G_1}\right) \end{pmatrix}^2}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} - \frac{\begin{pmatrix} \left(1 + (p_{\hat{V}} - p_V) \mathbf{K}^M_{1,1} + (p_{\hat{g}} - p_g) \mathbf{K}^M_{3,1} + (p_{\hat{G}} - p_G) \mathbf{K}^M_{4,1}\right) \Sigma^{M,VV} \\ + \left(1 + (p_{\hat{V}} - p_V) \mathbf{K}^M_{1,2} + (p_{\hat{g}} - p_g) \mathbf{K}^M_{3,2} + (p_{\hat{G}} - p_G) \mathbf{K}^M_{4,2}\right) \\ \times \left(p_V \rho_V \Sigma^{M,VV} + p_g \Sigma^{M,VG_1}\right) \end{pmatrix}^2}{\Sigma^{M,VV} + \tau_s^{-1}}$$

Recognizing that $\phi'\omega = CoV \left[R_{t+1}, \begin{bmatrix} V_{t+1} \\ G_{t+1} \end{bmatrix} \mid \mathcal{F}_t^M \right]$, we can rewrite the above more generally as:

$$Q = \frac{CoV \left[R_{t+1}, G_{t+1} \mid \mathcal{F}_t^M \right]^2}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} - \frac{CoV \left[R_{t+1}, V_{t+1} \mid \mathcal{F}_t^M \right]^2}{\Sigma^{M,VV} + \tau_s^{-1}}.$$

Proof of Proposition 5

In the special case that $\rho_V = 0$, it follows that the Kalman Gain, the steady-state market beliefs, and the $Q-$statistic for information acquisition satisfy:

$$\mathbf{K}^M = \begin{bmatrix} 0 & \frac{p_V \sigma_V^2}{p_V^2 \sigma_V^2 + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2} & 0 \\ 0 & \frac{p_N \sigma_N^2}{p_V^2 \sigma_V^2 + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2} & 0 \\ 0 & \frac{p_G \sigma_G^2}{p_V^2 \sigma_V^2 + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and:

$$\Sigma^M = \begin{bmatrix} \frac{p_N^2\sigma_N^2+p_G^2\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_V^2 & -\frac{p_V\sigma_V^2 p_N\sigma_N^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & -\frac{p_V\sigma_V^2 p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & 0 \\ -\frac{p_V\sigma_V^2 p_N\sigma_N^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & \frac{p_V^2\sigma_V^2+p_G^2\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_N^2 & -\frac{p_N\sigma_N^2 p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & 0 \\ -\frac{p_V\sigma_V^2 p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & -\frac{p_N\sigma_N^2 p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2} & \frac{p_V^2\sigma_V^2+p_N^2\sigma_N^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_G^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and:

$$\begin{aligned} Q \;=\; & \frac{\left(\frac{p_N^2\sigma_N^2+p_G^2\sigma_G^2+\frac{1}{R^f}p_V\sigma_V^2-p_g p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}p_g\left(p_V^2+p_N^2\frac{\sigma_N^2}{\sigma_V^2}\right)-p_V p_G\right)^2}{\frac{p_V^2\sigma_V^2+p_N^2\sigma_N^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_G^2+\tau_g^{-1}}\left(\frac{\sigma_V^2\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\right)^2 \\ & -\frac{\left(p_N^2\frac{\sigma_N^2}{\sigma_G^2}+p_G^2-\frac{p_N^2\sigma_N^2+p_G^2\sigma_G^2+\frac{1}{R^f}p_V\sigma_V^2-p_g p_G\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}p_g p_V p_G\right)^2}{\frac{p_N^2\sigma_N^2+p_G^2\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_V^2+\tau_s^{-1}}\left(\frac{\sigma_V^2\sigma_G^2}{p_V^2\sigma_V^2+p_N^2\sigma_N^2+p_G^2\sigma_G^2}\right)^2, \end{aligned}$$

respectively.

In a government-centric equilibrium, $p_V = 0$, and, from the market-clearing conditions, p_g and p_G satisfy:

$$\begin{aligned} p_g &= \frac{p_N\sigma_N}{1-\vartheta_{\hat{N}}}\sqrt{\frac{p_N^2\sigma_N^2}{p_N^2\sigma_N^2+p_G^2\sigma_G^2}\vartheta_{\hat{N}}^2}, \\ p_G &= \frac{1}{R^f}\left(1-\vartheta_{\hat{N}}\right)\frac{p_g p_N^2\sigma_N^2}{p_N^2\sigma_N^2+p_G^2\sigma_G^2}\sigma_G^2, \end{aligned}$$

from which follows that p_G is given by , $p_G^2 = x p_N^2\frac{\sigma_N^2}{\sigma_G^2}$ where x satisfies:

$$x\left(1+x\right)^3 = \left(\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3\right)^2,$$

where x is increasing in $\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3$. It then follows that Q reduces to:

$$Q = \left(\frac{\left(\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}\right)^2}{\sigma_G^2+\left(1+x\right)\tau_g^{-1}}\left(\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 p_N^2\sigma_N^2 - \frac{\sigma_V^4}{\sigma_V^2+\tau_s^{-1}}\left(1+x\right)^2\right)\left(\frac{\sigma_G^2}{1+x}\right)^2,$$

which suggests that, for $Q \geq 0$, it must be the case that:

$$p_N^2 > \bar{p}_N^2 = \frac{\sigma_V^4}{\sigma_N^2}\frac{\sigma_G^2+\left(1+x\right)\tau_g^{-1}}{\sigma_V^2+\tau_s^{-1}}\left(\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}\right)^2\left(\frac{1+x}{\sigma_G^2-R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2.$$

Furthermore, it is straightforward to compute that:

$$\phi'\Omega^M\phi = \sigma_V^2 + \sigma_D^2 + \sigma_G^2\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 p_N^2\sigma_N^2 + \frac{\left(1+\frac{1+x}{1-\vartheta_{\hat{N}}}\frac{1}{\sigma_G^2}x\right)^2}{1+x}p_N^2\sigma_N^2,$$

and therefore, from market-clearing, that p_N also satisfies:

$$\begin{aligned} 0 &= \left(\sigma_G^2\frac{\sigma_G^2+2(1+x)\tau_g^{-1}}{\sigma_G^2+(1+x)\tau_g^{-1}}\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 + \frac{\left(1+\frac{1+x}{1-\vartheta_{\hat{N}}}\frac{1}{\sigma_G^2}x\right)^2}{1+x}\right)\sigma_N^2 p_N^2 \\ &\quad -\frac{R^f}{1-\vartheta_{\hat{N}}}\frac{1+x}{\sigma_G^2+(1+x)\tau_g^{-1}}p_N + \sigma_V^2 + \sigma_D^2. \end{aligned}$$

It follows that p_N is given by the two roots of the above quadratic form:

$$p_N = \frac{1}{2\sigma_N^2 c}\frac{R^f}{1-\vartheta_{\hat{N}}} \pm \sqrt{\left(\frac{1}{2\sigma_N^2 c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{\sigma_N^2 c}},$$

where:

$$c = \sigma_G^2\frac{\sigma_G^2+2(1+x)\tau_g^{-1}}{\sigma_G^2+(1+x)\tau_g^{-1}}\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 + \frac{\left(1+\frac{1}{\sigma_G^2}\frac{1+x}{1-\vartheta_{\hat{N}}}x\right)^2}{1+x} \geq 0,$$

and $c = c\left(\vartheta_{\hat{N}}, R^f, \sigma_G\right)$. When p_N exists, one consequently has that $p_N > 0$. Selecting the less positive root, and recognizing that $Q \geq 0$ whenever $p_N \geq \bar{p}_N$, we can express this condition as:

$$\begin{aligned} &\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2}\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}} - \sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}\right) \qquad (1) \\ &\geq (1+x)\sqrt{\left(\sigma_G^2+(1+x)\tau_g^{-1}\right)\left(\frac{\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}}{\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2}. \end{aligned}$$

Notice that the LHS of equation (1) is always nonnegative, since it is $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2}p_N\sigma_N$, and that c and the RHS of equation (1) is independent of $\{\sigma_N, \sigma_V, \sigma_D\}$ since $x = x\left(\vartheta_{\hat{N}}, R^f, \sigma_G\right)$.

Since it is straightforward to compute that:

$$\begin{aligned} \frac{dp_N\sigma_N}{d\sigma_N} &= -\frac{1}{\sigma_N}\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\frac{\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}} - \frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}}{\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}} > 0, \\ \frac{dp_N\sigma_N}{d\sigma_D} &= \frac{\sigma_D}{c\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}} > 0, \end{aligned}$$

it follows that the LHS is increasing in σ_N and σ_D. Consequently, the existence condition for a government-centric equilibrium relaxes as σ_N and σ_D increase, and therefore a government-centric equilibrium is more likely to exist the higher are σ_N and σ_D.

Finally, with respect to σ_V, we recognize that, as $\sigma_V \to 0$, $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2} p_N \sigma_N \to \infty$, and consequently the LHS exceeds the RHS and $Q > 0$. Since $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2} p_N \sigma_N$ is continuous in V, it follows that a government-centric equilibrium exists within a neighborhood of $\sigma_V = 0$, and consequently exists for σ_V sufficiently small.

Proof of Proposition 6

We can express the conditional uncertainty about the deviation in the asset price from its fundamentals as:

$$\begin{aligned} F &= Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \mathcal{F}_t^M\right] \\ &= Var\left[\left(\phi - \begin{bmatrix}1\\0\\0\end{bmatrix}\right)' \varepsilon_{t+1}^M - p_{\hat{V}}\left(V_{t+2} - \rho_V \hat{V}_{t+1}^M\right) \mid \mathcal{F}_t^M\right] \\ &= \left(\phi - \begin{bmatrix}1\\0\\0\end{bmatrix}\right)' \Omega^M \left(\phi - \begin{bmatrix}1\\0\\0\end{bmatrix}\right) + p_{\hat{V}}^2\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) \\ &\quad -2p_{\hat{V}}\left(\phi - \begin{bmatrix}1\\0\\0\end{bmatrix}\right)' \begin{bmatrix}\rho_V \Sigma^{M,VV}\\ p_V\left(\rho_V^2\Sigma^{M,VV} + \sigma_V^2\right)\\ \rho_V \Sigma^{M,VG_1}\end{bmatrix}, \end{aligned}$$

which we can rewrite as:

$$\begin{aligned} Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \mathcal{F}_t^M\right] &= Var\left[P_{t+1} \mid \mathcal{F}_t^M\right] + p_{\hat{V}}^2\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) \\ &\quad -2p_{\hat{V}}\left(\phi - \begin{bmatrix}1\\0\\0\end{bmatrix}\right)' \begin{bmatrix}\rho_V \Sigma^{M,VV}\\ p_V\left(\rho_V^2\Sigma^{M,VV} + \sigma_V^2\right)\\ 0\end{bmatrix} \end{aligned}$$

In a government-centric equilibrium, $p_V = 0$ and there is no learning from prices about V_{t+1}, so $\Sigma^{M,VV}$ is exogenous to government intervention. Consequently, the above reduces to

$$Var\left[P_{t+1} - p_{\hat{V}} V_{t+2} \mid \mathcal{F}_t^M\right] = Var\left[P_{t+1} \mid \mathcal{F}_t^M\right] + p_{\hat{V}}^2\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) - 2\frac{\left(p_{\hat{V}}\rho_V \Sigma^{M,VV}\right)^2}{\Sigma^{M,VV} + \sigma_D^2},$$

and minimizing price deviation, is then equivalent to minimizing $Var\left[P_{t+1} \mid \mathcal{F}_t^M\right]$, or price volatility.