Online Appendix

Are High-Interest Loans Predatory?
Theory and Evidence from Payday Lending

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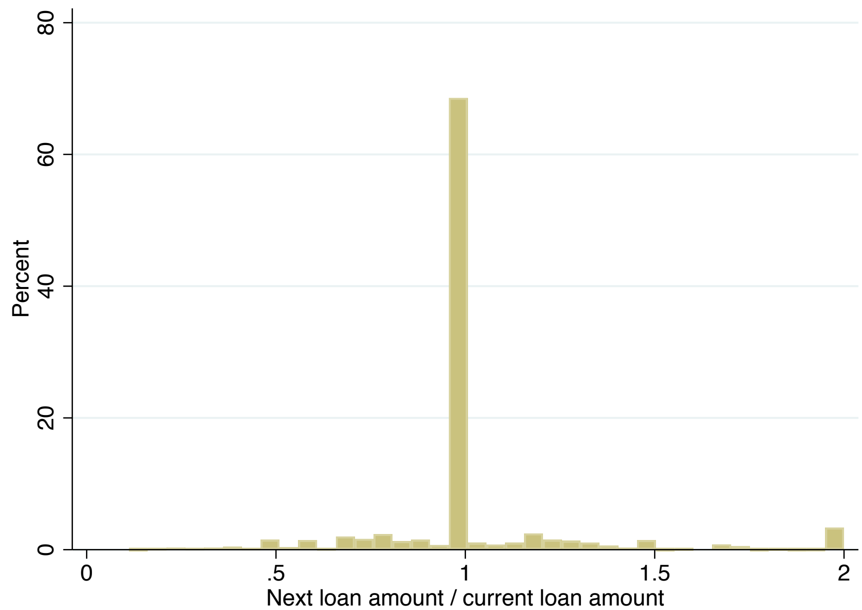
A Data Appendix

Figure A1: **Ratio of Next Loan Size to Current Loan Size**



Notes: For a random sample of payday loans disbursed by the Lender nationwide in 2017, this figure presents the ratio of the borrower's next loan size to the current loan size, for loans taken out within eight weeks of each other.

Table A1: **Descriptive Statistics**

	(1) Data source	(2) Mean	(3) Standard deviation	(4) Minimum value	(5) Maximum value
Loans in past six months	Lender	5.35	2.94	0	15
Annual income ($000s)	Lender	34.0	21.1	1	212
Internal credit score	Lender	862	122	0	997
Pay cycle length (days)	Lender	16.0	7.7	7	30
Loan length (days)	Lender	17.3	5.9	14	35
Loan amount ($)	Lender	373	161	50	600
Took survey in store	Survey	0.97	0.16	0	1
Predicted borrowing probability	Survey	0.70	0.35	0	1
Predicted borrowing probability with incentive	Survey	0.50	0.39	0	1
Valuation of incentive	Survey	52.2	44.8	0	155
Valuation of coin flip	Survey	42.2	33.0	0	155
“Very much” want motivation	Survey	0.54	0.50	0	1
Took out loans “more often than expected”	Survey	0.36	0.48	0	1
Borrowing restrictions “good” for me	Survey	0.28	0.45	0	1
Reborrowed over next eight weeks	Veritec	0.73	0.45	0	1
Reborrowed from Lender over next eight weeks	Lender	0.73	0.44	0	1

Notes: This table presents descriptive statistics for the sample of borrowers with valid survey responses. The sample size is 784 for internal credit score and 1,205 for all other variables.

Table A2: **Balance**

	(1) Control (SE)	(2) Incentive (SE)	(3) Difference (SE)
Loans in past six months	5.30	5.44	-0.14
	(0.11)	(0.13)	(0.17)
Annual income ($000s)	33.7	34.0	-0.35
	(0.8)	(0.9)	(1.23)
Internal credit score	865	858	7.14
	(4)	(5)	(5.76)
Pay cycle length (days)	15.8	16.3	-0.52
	(0.3)	(0.3)	(0.45)
Loan length (days)	17.26	17.48	-0.22
	(0.24)	(0.25)	(0.35)
Loan amount ($)	373	370	3.69
	(6)	(7)	(9.39)
Took survey in store	0.97	0.98	-0.01
	(0.01)	(0.01)	(0.01)
Predicted borrowing probability	0.69	0.71	-0.02
	(0.01)	(0.01)	(0.02)
Predicted borrowing probability with incentive	0.48	0.51	-0.03
	(0.02)	(0.02)	(0.02)
Valuation of incentive	54.6	50.1	4.51
	(1.8)	(1.9)	(2.61)
Valuation of coin flip	42.0	42.8	-0.80
	(1.3)	(1.4)	(1.93)
"Very much" want motivation	0.56	0.52	0.04
	(0.02)	(0.02)	(0.03)
Took out loans "more often than expected"	0.33	0.40	-0.07
	(0.02)	(0.02)	(0.03)
Borrowing restrictions "good" for me	0.30	0.27	0.03
	(0.02)	(0.02)	(0.03)
N	633	544	
F-test of joint significance (p-value)			0.20
F-test, number of observations			1,177

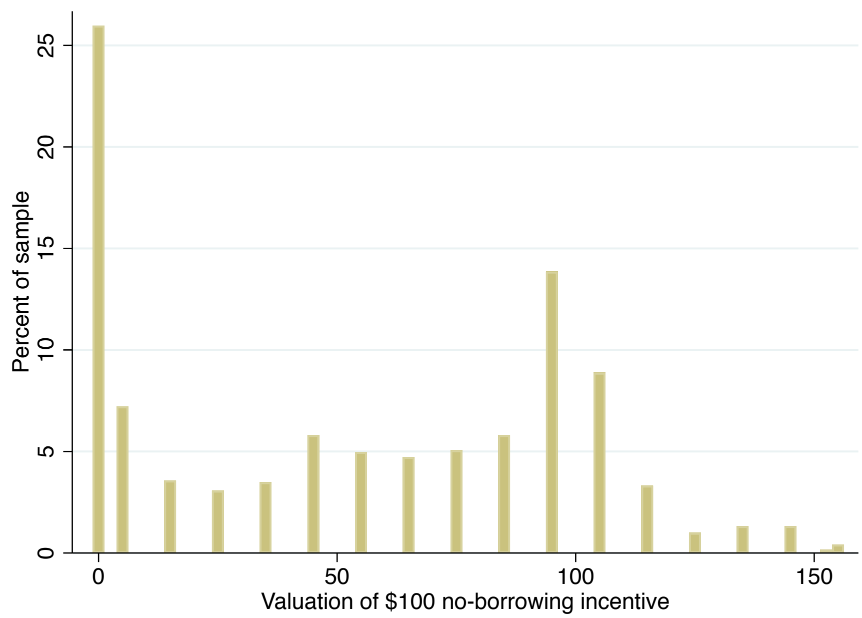
Notes: This table presents means and differences in means of baseline and survey variables for the Control and Incentive groups, with standard errors in parentheses. The data exclude 28 observations that were not assigned to the Control or Incentive groups.

Table A3: **Refusal and Sample Restrictions**

Sample restriction	N
Customers on survey days	13,191
Consented or declined	2,243
Consented	2,236
Completed survey	2,122
Matched to Lender data	1,943
Understood no-borrowing incentive	1,628
Passed attention check	1,428
Consistent MPL choices	1,392
Valuation of incentive < \$160	1,205

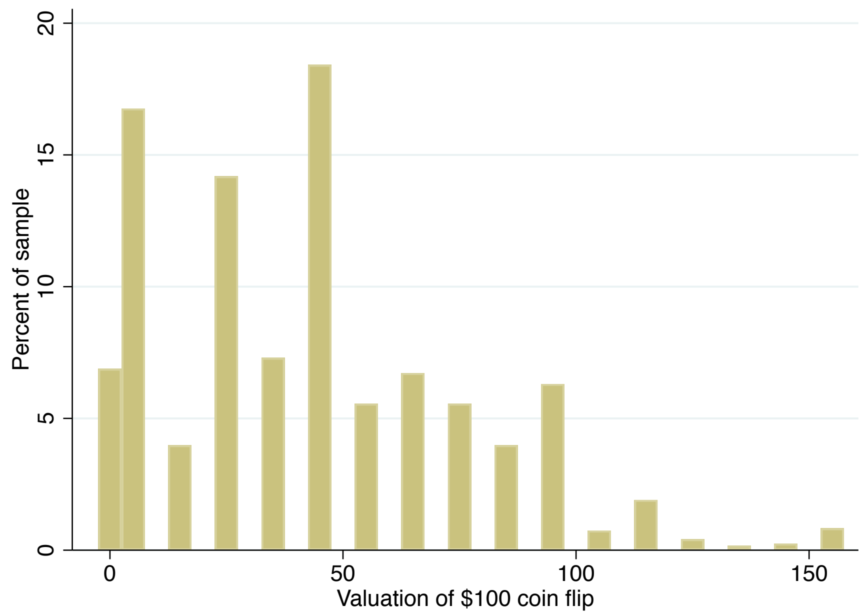
Notes: This table presents sample sizes after refusals and sample restrictions. "Customers on survey days" means all customers who got a loan from a Lender's store on a day when the survey was available in that store.

Figure A2: **Distribution of Valuations of the No-Borrowing Incentive**



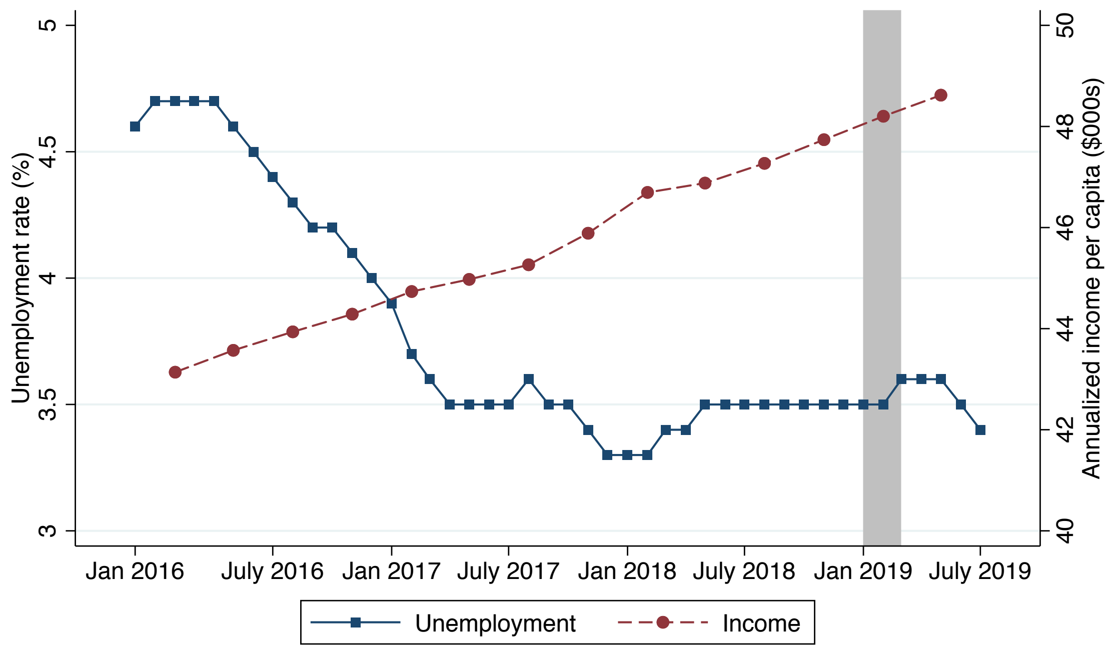
Notes: This figure presents the distribution of valuations of the \$100 no-borrowing incentive, as revealed on the survey.

Figure A3: **Distribution of Valuations of the $100 Coin Flip**



Notes: This figure presents the distribution of valuations of the Flip a Coin for $100 reward, as revealed on the survey.

Figure A4: **Indiana Macroeconomic Trends Before and After Survey**



Notes: This figure presents the unemployment rate and average annualized income in Indiana during the study period and for the three years before. Unemployment rate is from the Federal Reserve Bank of St. Louis (2019). Income is in nominal dollars and is from BEA (2019). The grey shaded area illustrates that all surveys were taken between January and March 2019.

Table A4: **Descriptive Statistics for Expert Survey**

Number of respondents	103
Percent academic economists	68%
Opinions about borrower decision making	
Think the average borrower underestimates reborrowing	78%
Average belief about borrowers' predicted reborrowing probability	40%
Think that the average borrower wants extra motivation to avoid borrowing	56%
Average belief about borrowers' perceived present bias parameter $\widetilde{\beta}$	0.86
Average certainty of opinion about borrower decision-making $(0 = \text{not at all}, 1 = \text{extremely})$	0.50
Opinions about effects of payday lending regulation on consumers	
Think prohibiting payday lending is good	56%
Think a rollover restriction with "cooling off period" is good	50%
Think limiting loan size to 5% of income is good	41%
Average certainty about effects of regulation $(0 = \text{not at all}, 1 = \text{extremely})$	0.44

Notes: This table presents descriptive statistics from a survey in which we asked academic and non-academic payday lending experts to predict the results of our study before it was released.

B Predicted Borrowing Data

B.1 Rounding

Figure A5 plots the distribution of predicted reborrowing probabilities without the no-borrowing incentive. Figure A6 plots the distribution of predicted reborrowing probabilities with the incentive. Both figures demonstrate excess mass at 0 percent, 50 percent, and 100 percent, suggesting that survey respondents may gravitate towards round numbers. Since all of our results on predicted reborrowing use averages across respondents, rounding affects our results if and only if it affects the average.

To examine how rounding may affect our results, we conduct an illustrative exercise where we estimate what the counterfactual distribution of predicted reborrowing probabilities may look like in the absence of rounding. We suppose that survey respondents who would answer 0.1 or 0.2 in the absence of rounding, may instead round to 0. Similarly, respondents who would answer 0.8 or 0.9 would round to 1, and respondents who would answer 0.3, 0.4, 0.6, or 0.7 would round to 0.5. Let μ denote the probability that a respondent chooses to round their answer.

We estimate μ by assuming that in the absence of rounding, the number of respondents who would choose to answer a focal number would equal the average number of respondents who chose to answer a non-focal number in the same rounding bin. For example, we assume that the number of respondents who would answer 0 would equal the average number of respondents who chose 0.1 or 0.2. We can then calculate the number of "excess" respondents who chose the focal number, and divide by the number of total respondents in a rounding bin to estimate μ, which we estimate to be $\hat{\mu} = 0.51$. For each rounding bin, we can then estimate the number of respondents who chose to round to the focal number, and redistribute them to recover the counterfactual distribution without rounding.

We plot the distribution of predicted reborrowing probabilities, correcting for rounding, in A7. The new average probability of reborrowing after correcting for rounding is 68 percent, compared to the 70 percent we found without correcting for rounding. Thus, rounding does not seem to substantially affect our results.

B.2 Survey response noise

Figure A8 presents a binned scatterplot of participants' predicted probability of reborrowing versus the actual proportion of customers assigned to our Control group who took out an additional loan. We see that borrowers who reported a higher probability of reborrowing were substantially more likely to actually borrow.

The relationship between predicted and actual borrowing probability is attenuated relative to a 45-degree line, consistent with the expected effects of measurement error due to noisy survey responses. Such noise could be driven by accidentally clicking the wrong numbers or cognitive difficulties in articulating probabilities. To illustrate how measurement error can attenuate the relationship between predicted and actual probabilities, we run a simple simulation where individuals

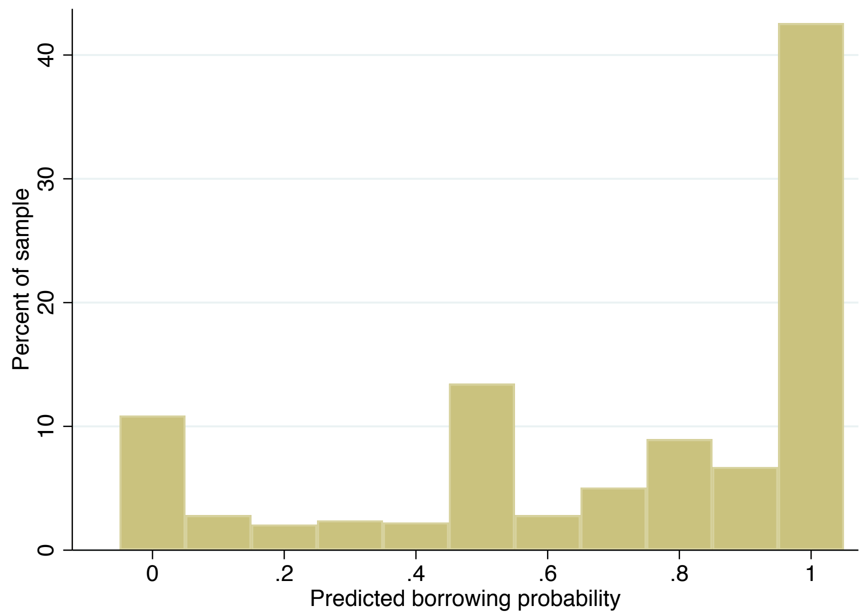
report a predicted probability with noise.

We first draw 10,000 true borrowing probabilities p from a $Beta(2.9, 1)$ distribution. The average of simulated probabilities p thus equals the actual probability of reborrowing we observe in our sample: 74 percent. To simulate measurement error, we follow the procedure used in Mueller, Spinnewijn, and Topa (2019): given a true probability p and a noise parameter d, individuals report a predicted probability $\hat{p} = p + \epsilon$, where $\epsilon \sim U(-d, d)$ if $p - d \geq 0$ and $p + d \leq 1$. If $p - d < 0$, then $\epsilon \sim U(-p, d)$, with a mass-point at $-p$ such that $E[\epsilon] = 0$. Similarly, if $p + d > 1$, then $\epsilon \sim U(-d, 1 - p)$, with a mass-point at $1 - p$ such that $E[\epsilon] = 0$. Panel (a) of Figure A9 presents the results of our simulations with $d = 0.5$.

We also account for rounding bias when individuals report $\hat{p}$. If $\hat{p} \in [0, 0.25)$, then with 50 percent probability, they report $\hat{p} = 0$. Similarly, if $\hat{p} \in [0.25, 0.75)$ or $\hat{p} \in [0.75, 1]$, they report $\hat{p} = 0.5$ or $\hat{p} = 1$ with 50 percent probability, respectively. Panel (b) plots a simulated binscatter plot with both measurement error where $d = 0.5$ and rounding bias.

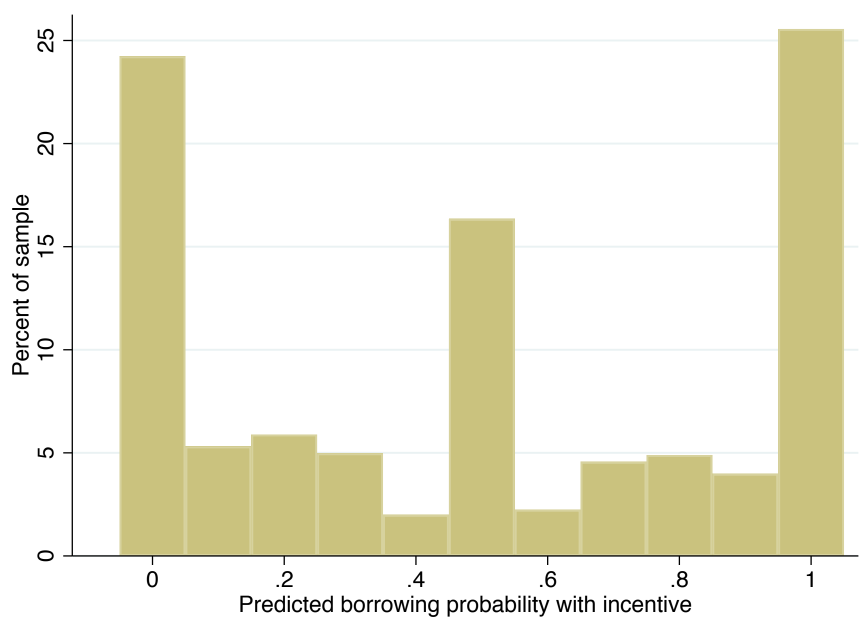
Neither mean-zero measurement error or rounding bias substantially affect our results: the actual average of simulated probabilities is 74 percent and the average of simulated probabilities with measurement error and rounding is 74.3 percent.

Figure A5: **Distribution of Predicted Borrowing Probability**



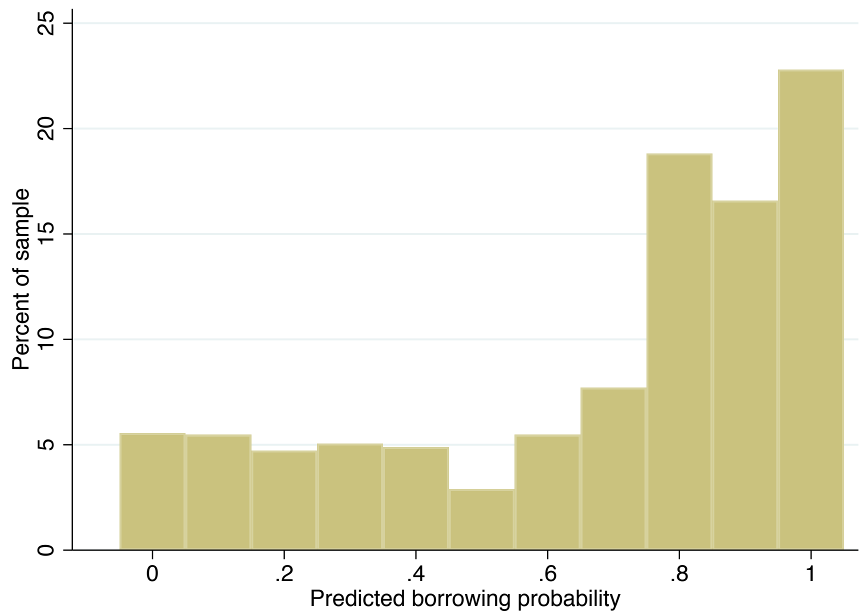
Notes: This figure presents the distribution of answers to the following question: "What do you think is the chance that you will get another payday loan from any lender before [eight weeks from now]?"

Figure A6: **Distribution of Predicted Borrowing Probability with No-Borrowing Incentive**



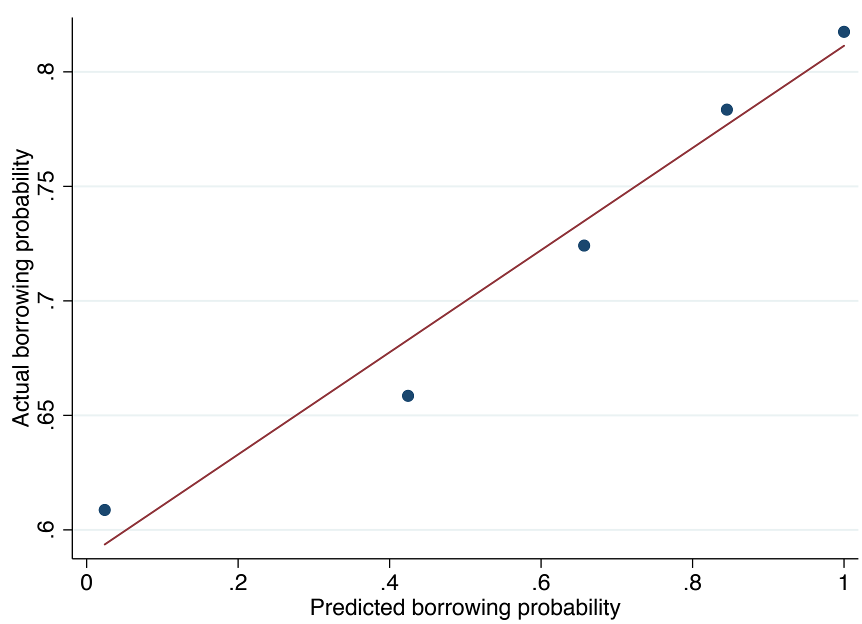
Notes: This figure presents the distribution of answers to the following question: **"If you are selected for $100 If You Are Debt-Free**, what is the chance that you would get another payday loan from any lender before [eight weeks from now]?"

Figure A7: **Distribution of Debiased Predicted Borrowing Probability**



Notes: This figure presents the counterfactual (without rounding) distribution of responses to the following question: "What do you think is the chance that you will get another payday loan from any lender before [eight weeks from now]?"

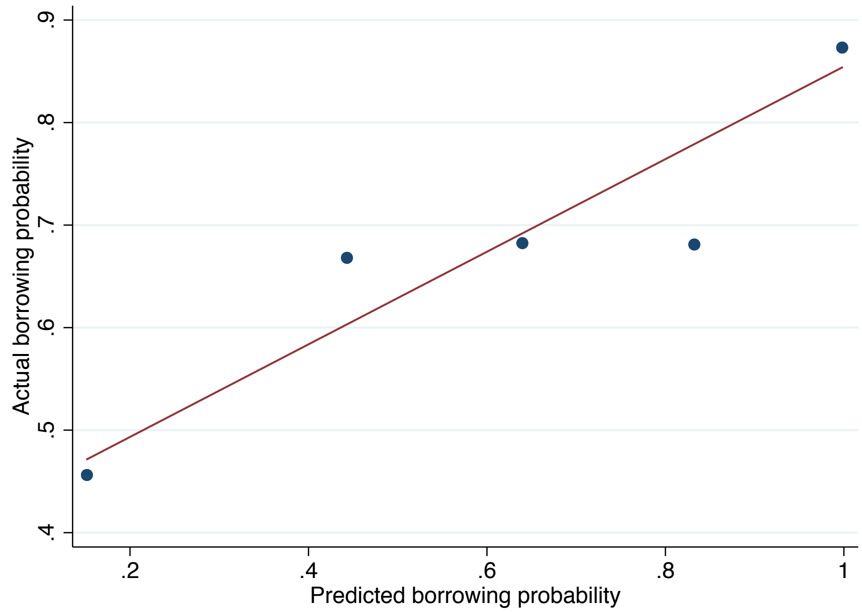
Figure A8: **Predicted versus Actual Borrowing**



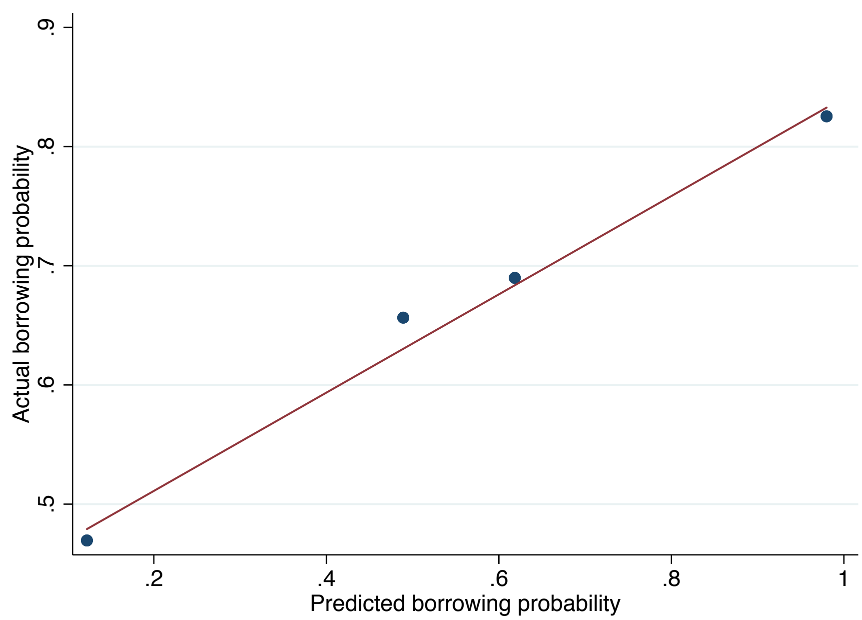
Notes: This figure presents a binned scatterplot of actual versus predicted probability of getting another payday loan in the next eight weeks after the survey, for the Control group.

Figure A9: **Predicted versus Actual Borrowing (Simulations)**

(a) **Simulations with Measurement Error**



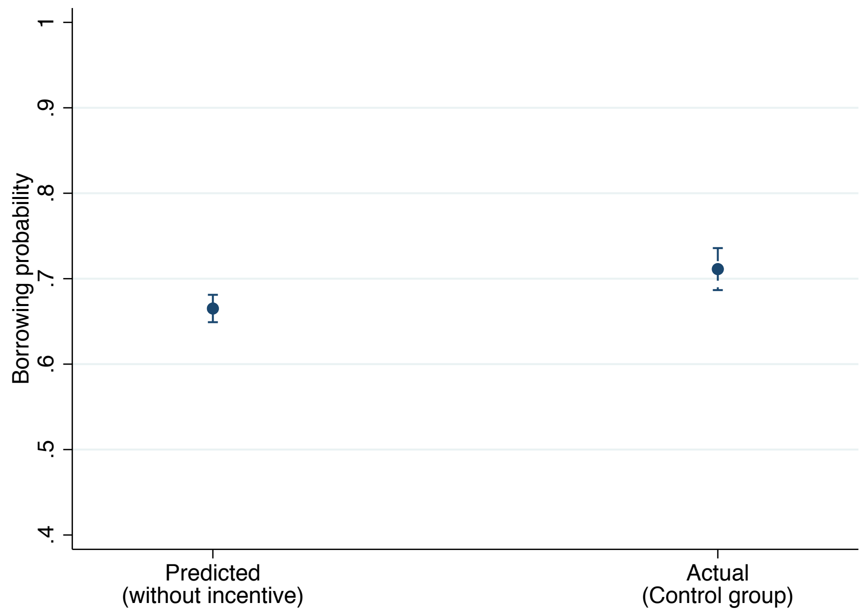
(b) **Simulations with Measurement Error and Rounding Bias**



Notes: These figures present the effects of measurement error and rounding bias on predicted versus actual borrowing probabilities. Panel (a) plots a binned scatterplot of predicted versus actual borrowing probability with mean zero measurement error. Panel (b) includes mean zero measurement error and rounding bias.

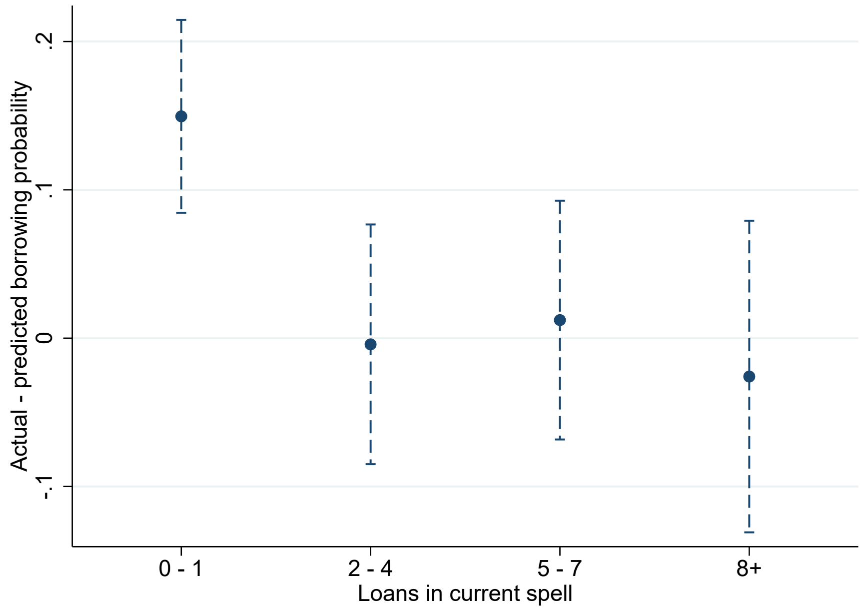
C Additional Results for Section 6

Figure A10: **Predicted and Actual Borrowing Without Pre-Registered Exclusion Restrictions**



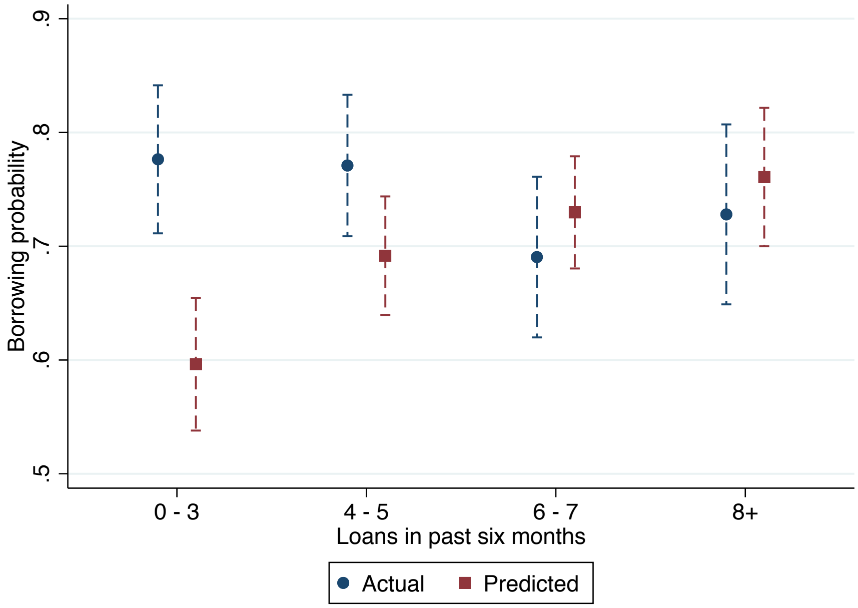
Notes: The data includes all participants, including those whom we pre-registered to exclude. The left spike presents the average predicted probability of getting another payday loan in the next eight weeks without the no-borrowing incentive. The right spike presents the actual probability of getting another payday loan in the next eight weeks for the Control group, which did not receive the no-borrowing incentive. Error bars represent 95 percent condence intervals.

Figure A11: **Heterogeneity in Misprediction by Loan in Spell**



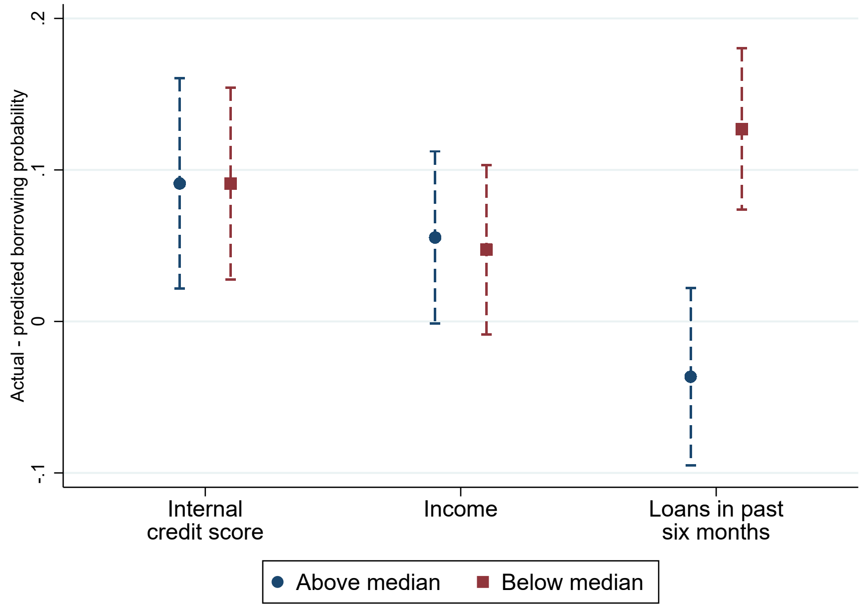
Notes: This figure presents the actual borrowing probability minus the average predicted borrowing probability for subgroups defined by the number of consecutive loans taken out from the Lender before the survey date. "Consecutive loans" are loans taken out within eight weeks of each other. The figure includes only the Control group. Error bars represent 95 percent confidence intervals.

Figure A12: **Heterogeneity in Misprediction by Experience**



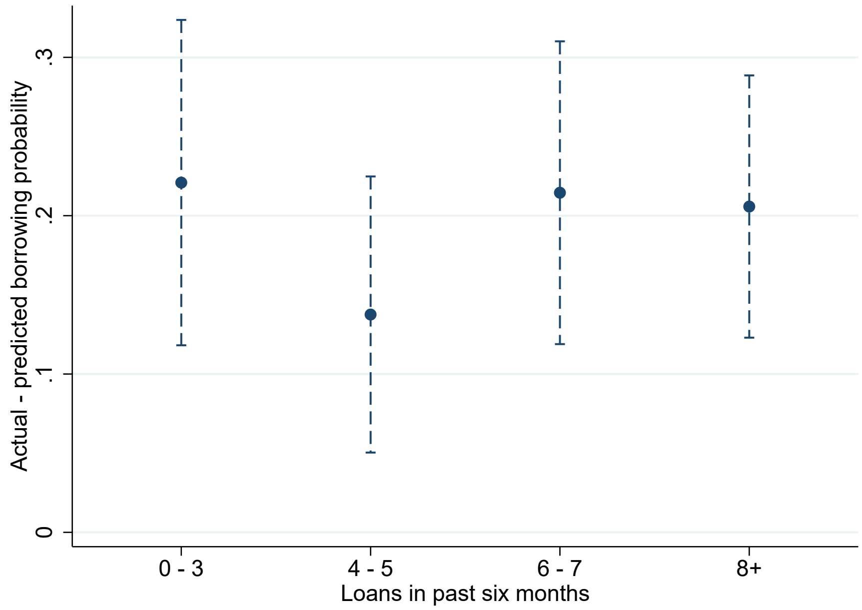
Notes: This figure presents the actual borrowing probability and the average predicted borrowing probability for subgroups defined by the number of loans taken out from the Lender in the six months before taking the survey. The figure includes only the Control group. Error bars represent 95 percent confidence intervals.

Figure A13: **Heterogeneity in Misprediction**



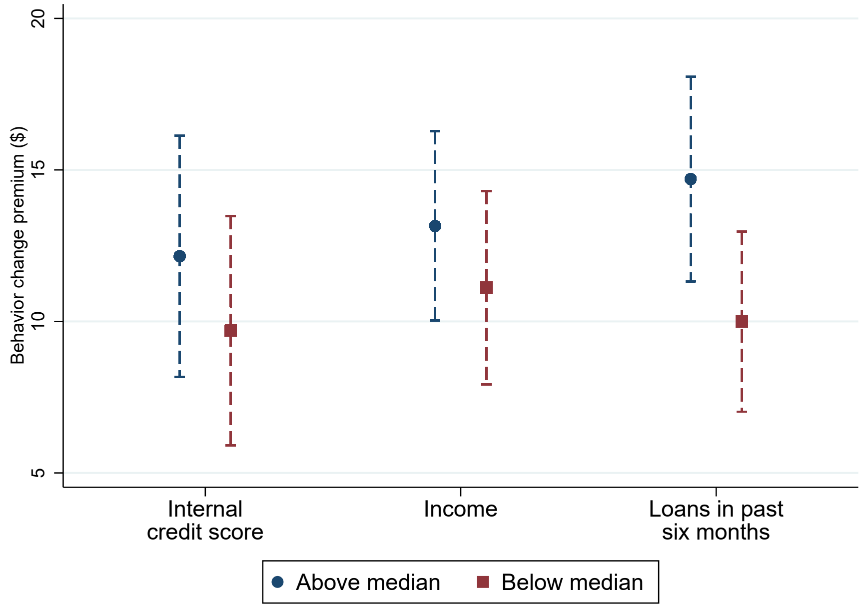
Notes: This figure presents the actual borrowing probability minus the average predicted borrowing probability for borrowers with above- versus below-median internal credit score, income, and number of loans taken out from the Lender in the six months before taking the survey. The figure includes only the Control group. Error bars represent 95 percent confidence intervals.

Figure A14: **Misprediction by Experience in Incentive Group**



Notes: This figure presents the actual borrowing probability minus the average predicted borrowing probability for subgroups defined by the number of loans taken out from the Lender in the six months before taking the survey. This figure includes only the Incentive group. Error bars represent 95 percent confidence intervals.

Figure A15: **Heterogeneity in Behavior Change Premium by Borrower Characteristics**



Notes: This figure presents the behavior change premium for borrowers with above- versus below-median internal credit score, income, and number of loans taken out from the Lender in the six months before taking the survey. The behavior change premium equals $w(100) - w^*(100)$, the valuation of the $100 no-borrowing incentive minus the modeled valuation for a risk-neutral and time-consistent borrower. Error bars represent 95 percent confidence intervals.

D Formal Statement of Section 6.2 Envelope Theorem Arguments

Appendix D.1 summarizes the key results using simplified notation. Appendices D.2 and D.3 provide the formal results.

D.1 Summary

Consider a time-consistent borrower determining her change in valuation $dw(b)$ from a marginal incentive change db. Let $\bar{m}_1$ and $\bar{m}_0$, respectively, denote the expected marginal utilities of money (for the time when the experimental payments are made) across states of the world in which the person does and does not borrow. The borrower predicts that she will avoid borrowing with probability $1-\tilde{\mu}$, and the expected marginal utility in that state of the world is $\bar{m}_0$, so the Envelope Theorem implies that utility from the change db is $(1-\tilde{\mu})db\bar{m}_0$. Similarly, the expected utility from dw Money for Sure is $dw\left(\tilde{\mu}\bar{m}_1 + (1-\tilde{\mu})\bar{m}_0\right)$.

Thus, the valuation $dw(b)$ of a marginal incentive change db satisfies

$$\underbrace{dw(b)\left(\tilde{\mu}\bar{m}_1 + (1-\tilde{\mu})\bar{m}_0\right)}_{\text{expected utility from Money for Sure}} = \underbrace{(1-\tilde{\mu})db\bar{m}_0}_{\text{expected utility from incentive}}. \tag{11}$$

If the borrower is risk neutral over income, then $\bar{m}_0 = \bar{m}_1$, and $dw(b) = (1-\tilde{\mu})db$. In practice, we expect $\bar{m}_0 \leq \bar{m}_1$, both because the income from a non-marginal incentive reduces marginal utility and because people have higher marginal utility of income in the states of the world where they need to borrow. This implies

$$dw(b) \leq (1-\tilde{\mu})db. \tag{12}$$

To determine a borrower's valuation of a non-marginal \$100 no-borrowing incentive, we integrate over Equation (12) as formalized in Appendix D.2. Assuming that $\tilde{\mu}$ is locally linear in the incentive over this range, we show in Appendix D.3 that borrowers who perceive themselves to be time consistent must have

$$w(b) \leq w^*(b) := b - b\frac{\tilde{\mu}(0) + \tilde{\mu}(b)}{2}. \tag{13}$$

With constant marginal utility, this holds with equality, and the right-hand side is the valuation we derived graphically on Figure 1: $b - ABCD$. Because the bound uses subjective expectations $\tilde{\mu}$, it is valid for borrowers who mispredict borrowing for any reason, including if they unexpectedly forget about the no-borrowing incentive.

Now consider borrowers who perceive themselves to be time *in*consistent. The standard Envelope Theorem logic does not apply: borrowers who perceive themselves to be time inconsistent believe that their future behavior will not optimize current utility, so their valuation of the incentive includes the current utility benefits from behavior change. The perception that one will "overborrow" ("underborrow") in the future relative to current preferences will increase (decrease)

valuations of the no-borrowing incentive.[34]

D.2 Envelope Theorem with general choice sets

Formally, we conceptualize the time consistent borrower's problem as follows. There is a space Ω of states of the world equipped with a sigma algebra Σ_Ω. A state of the world can constitute a sequence of liquidity, income, and other types of shocks, as well as signals about future liquidity, income, and other consequential outcomes. There is also a set of possible actions A, equipped with a sigma algebra Σ_A, which can constitute sequences of borrowing, repaying, and any other consumption and savings decisions. A feasible plan is a measurable function $\mathbf{a} : (\Omega, \Sigma_\Omega) \to (A, \Sigma_A)$ that prescribes a sequence of behavior for a realized state of the world, and we let $\mathcal{A}$ denote the set of all feasible plans. Let the experimental incentives be $(w, b) \in [0, \bar{b}] \times [0, \bar{w}]$.

To be clear, this formulation allows for arbitrarily dynamic decisions. For example, let $t = 0$ denote the period in which the borrower takes the study, and let T denote the end of the game. A state of the world $\omega \in \Omega$ can constitute a sequence of realizations $\omega = (\omega_{1,}, \ldots, \omega_T)$. An action $a = (a_1, \ldots a_T)$ constitutes a sequences of choices in each period. At each time period t, the choice-set A_t may be a function of the history $h_t = (\omega_{1,}, \ldots, \omega_{t-1}, a_1, \ldots, a_{t-1})$. Then $\mathbf{a} \in \mathcal{A}$ is a plan for a choice of a_t after each realized history h_t. The utility function u can corresponded to the discounted sum $\sum_{t=1}^{T} \delta^t F_t(a_t, h_t, \omega_t, b, w)$ of flow utility functions F_t that can depend arbitrarily on the states of the world and the decisions made by the borrower. In this formulation we can also remain agnostic about which time period corresponds to the 8-week mark after the start of the study, and which time period corresponds to the 12-week mark at which the experimental incentives are delivered. A time period could be an hour, a day, a week, or anything else. Because borrowers are time-consistent and thus do not wish to revise their state-contingent plans, the dynamic decisions can be represented by a single static choice of a state-contingent plan.

For a plan $\mathbf{a}$, let $\Omega_R(\mathbf{a})$ denote the subset of states for which the borrow is "debt-free" for 8 weeks after the start of the experiment. Let $\mathbf{1}(\omega \in \Omega_D(\mathbf{a})) \in \{0, 1\}$ be an indicator for whether a state belongs $\Omega_R(\mathbf{a})$. Expected utility given a plan $\mathbf{a}$ and experimental incentives (w, b) is

$$U(\mathbf{a}, w, b) = \int_\Omega u(\mathbf{a}(\omega), y; \omega) d\nu(\omega)$$

where $y = w + b\mathbf{1}(\omega \in \Omega_R(\mathbf{a}))$, ν is the measure on (Ω, Σ_Ω), and u is realized utility for each plan and sequence of actions. Our main assumption is that $u(a, y; \omega)$ is continuously differentiable in y for all a and ω, and that the derivative with respect to y is bounded: $\sup_{a,y,\omega} |\frac{\partial}{\partial w} u(a, y; \omega)| < \infty$. This ensures that U is equidifferentiable in w and b on compact set $[0, \bar{b}] \times [0, \bar{w}]$.[35] Theorem 3 of Milgrom and Segal (2002) then ensures that $V(w, b) = \sup_{\mathbf{a} \in \mathcal{A}} U(\mathbf{a}, w, b)$ is continuously differentiable in w, b.

For incentives (w, b), let $\mathbf{a}^*_{w,b}$ be anoptimal plan chosen by the borrower, satisfying $U(\mathbf{a}^*_{w,b}, w, b) =$

[34] If people understate their true predicted borrowing probability $\tilde{\mu}(b)$ on the survey, then the valuation bound we calculate is higher than the "correct" valuation bound, which biases against detecting perceived time inconsistency.

[35] See Milgrom and Segal (2002), p 587 for the definition of equidifferentiability.

$V(w,b)$. Note that optimal plans need not be unique, and we use $\mathbf{a}^*$ to denote the borrower's selection.

Set $\tilde{\mu}(w,b) := \nu\left(\Omega_R(\mathbf{a}^*_{w,b})\right)$. Define $\bar{m}_0(w,b) := E\left[\frac{\partial}{\partial y}u(\mathbf{a}^*_{w,b}(\omega),y;\omega)|_{y=w+b}|\omega \in \Omega_R(\mathbf{a}^*_{w,b})\right]$ to be the average marginal utility from income in states of the world in which the borrower does not reborrow in the 8-week period. Define $\bar{m}_1(w,b)$ analogously to be the average marginal utility from income in states of the world in which the borrower does reborrow in the 8-week period.

Theorem 1 of Milgrom and Segal (2002) then gives the following:

$$\begin{aligned}
\frac{\partial}{\partial b}V(w,b) &= \frac{\partial}{\partial b}U(\mathbf{a},w,b) \\
&= \int_{\Omega_R(\mathbf{a}^*_{w,b})} \frac{\partial}{\partial y}u(\mathbf{a}^*_{w,b}(\omega),y;\omega)|_{y=w+b}d\nu(\omega) \\
&= \bar{m}_0(w,b)(1-\tilde{\mu}(w,b)) \qquad (14) \\
\frac{\partial}{\partial w}V(w,b) &= \frac{\partial}{\partial w}U(\mathbf{a},w,b) \\
&= \int_{\Omega_R(\mathbf{a}^*_{w,b})} \frac{\partial}{\partial y}u(\mathbf{a}^*_{w,b}(\omega),y;\omega)|_{y=w+b}d\nu(\omega) + \int_{\Omega_R(\mathbf{a}^*_{w,b})^c} \frac{\partial}{\partial y}u(\mathbf{a}^*_{w,b}(\omega),y;\omega)|_{y=w}d\nu(\omega) \\
&= \bar{m}_0(w,b)(1-\tilde{\mu}(w,b)) + \bar{m}_1(w,b)\tilde{\mu}(w,b) \qquad (15)
\end{aligned}$$

Now define $w(b)$ to satisfy $V(w(b),0) = V(0,b)$, which is differentiable by the Implicit Function Theorem. From (14) and (15) above, we have

$$w'(b) = \frac{\frac{\partial}{db}V(w(b),b)}{\frac{\partial}{dw}V(w(b),b)} = \frac{\bar{m}_0(w(b),b)(1-\tilde{\mu}(w(b),b))}{\bar{m}_0(w(b),b)(1-\tilde{\mu}(w(b),b)) + \bar{m}_1(w(b),b)\tilde{\mu}(w(b),b)}$$

Under the assumption that $\bar{m}_0 \leq \bar{m}_1$, which holds in our microfounded structural model, we then have that $w'(b) \leq (1-\tilde{\mu}(w(b),b))$. Note that there are two general economic reasons for $\bar{m}_0 \leq \bar{m}_1$. First, y is mechanically higher when the borrower does not reborrow. Second, carrying debt increases marginal utility from money.

Moreover, under the assumption that guaranteed future income in 12 weeks encourages borrowers to continue reborrowing over the first 8 weeks, we have that $\tilde{\mu}(w(b),b) \geq \tilde{\mu}(0,b)$, and thus that

$$w'(b) \leq (1-\tilde{\mu}(0,b))$$

from which it follows that

$$w(b) \leq \int_{x=0}^{b}(1-\tilde{\mu}(0,x))dx. \qquad (16)$$

D.3 Approximations to $w(b)$

We define γ to be the size of the no-borrowing incentive used in the experiment. To be concise, we now write $\tilde{\mu}$ as a function of b only, assuming that it is evaluated at $w = 0$. If $\tilde{\mu}$ is (weakly)

concave in b, then

$$\begin{aligned}\int_{b=0}^{\gamma}\tilde{\mu}(b)db &= \gamma\int_{t=0}^{1}\tilde{\mu}(t\gamma)dt\\ &\geq \gamma\int_{t=0}^{1}\left[(1-t)\tilde{\mu}(0)+t\tilde{\mu}(\gamma)\right]dt\\ &= \gamma\tilde{\mu}(0)+\gamma\int_{t=0}^{1}t(\tilde{\mu}(\gamma)-\tilde{\mu}(0))dt\\ &= \gamma\left[\tilde{\mu}(0)+\frac{1}{2}(\tilde{\mu}(\gamma)-\tilde{\mu}(0))\right]\end{aligned}$$

with equality holding when $\tilde{\mu}$ is linear in b on $[0,\bar{\gamma}]$, since in that case $\tilde{\mu}(t\gamma)=(1-t)\tilde{\mu}(0)+t\tilde{\mu}(\gamma)$. Thus, when w is (weakly) concave,

$$w(\gamma)\leq\int_{b=0}^{\gamma}(1-\tilde{\mu}(0,b))db\leq\gamma\left[1-\tilde{\mu}(0)+\frac{1}{2}(\tilde{\mu}(0)-\tilde{\mu}(\gamma))\right].$$

Next, suppose that $\tilde{\mu}$ is convex, with $\tilde{\mu}'(\gamma)=k\tilde{\mu}'(0)$ for $k\in(0,1)$. Make the quadratic approximation that terms of order $\tilde{\mu}'''$ and higher are negligible. Then $\tilde{\mu}'(\gamma)=\tilde{\mu}'(0)+\gamma\tilde{\mu}''$, and thus $\tilde{\mu}''=\frac{\tilde{\mu}'(\gamma)-\tilde{\mu}'(0)}{\gamma}=\frac{k-1}{\gamma}\tilde{\mu}'(0)$. Moreover,

$$\begin{aligned}\tilde{\mu}(\gamma)-\tilde{\mu}(0) &= \tilde{\mu}'(0)\gamma+\tilde{\mu}''(0)\gamma^2/2\\ &= \tilde{\mu}'(0)\gamma\left(1+\frac{k-1}{2}\right)\\ &= \tilde{\mu}'(0)\gamma(k+1)/2\end{aligned}$$

and thus $\tilde{\mu}'(0)=2\frac{\tilde{\mu}(\gamma)-\tilde{\mu}(0)}{\gamma(k+1)}$

Then the bound on surplus is given by

$$\begin{aligned}\int_{b=0}^{\gamma}(1-\tilde{\mu}(b))db &= (1-\tilde{\mu}(0))\gamma-\tilde{\mu}'(0)\frac{\gamma^2}{2}-\tilde{\mu}''\frac{\gamma^3}{6}\\ &= (1-\tilde{\mu}(0))\gamma-\tilde{\mu}'(0)\frac{\gamma^2}{2}+\tilde{\mu}'(0)\frac{1-k}{6}\gamma^2\\ &= (1-\tilde{\mu}(0))\gamma-2\frac{\tilde{\mu}(\gamma)-\tilde{\mu}(0)}{\gamma(k+1)}\left(\frac{\gamma^2}{2}-\frac{1-k}{6}\gamma^2\right)\\ &= (1-\tilde{\mu}(0))\gamma-(\tilde{\mu}(\gamma)-\tilde{\mu}(0))\gamma\left(\frac{1}{k+1}-\frac{1-k}{3}\right)\\ &= (1-\tilde{\mu}(0))\gamma+(\tilde{\mu}(0)-\tilde{\mu}(\gamma))\gamma\left(\frac{2+k}{3+3k}\right)\end{aligned}$$

When $k=1/2$, we have $\frac{2+k}{3+3k}=0.56$. When $k=0.25$, we have $\frac{2+k}{3+3k}=0.6$.

E Additional Results and Proofs for Section 7

In our simplified exposition in Section 7, the only repayment cost shock was θ. More generally, we now suppose that the cost functions are now given by $k(x, \theta, \eta)$ and $\tilde{C}(x, \eta)$, where η is a correlated shock distributed according to G. We define $\alpha = E[\tilde{C}''(0, \eta)/\tilde{C}'(0, \eta)]$, and $\rho = \alpha(l + p)$. We use γ instead of \$100 to refer to the amount of the no-borrowing incentive, which helps to clarify where that enters the equations.

To establish the results in the body of the paper, we assume that $\frac{k(l+p,\theta,\eta)-k(p,\theta,\eta)}{\tilde{C}(x_1,\eta)-\tilde{C}(x_2,\eta)} \perp \tilde{C}'(x, \eta) \forall \eta, x_1, x_2$. This stronger assumption guarantees that period 1 decisions relate to period 2 marginal utility only through how they affect period 2 debt.

To identify perceived present focus while allowing for risk aversion, we need to consider how predicted reborrowing $\tilde{\mu}$ depends on both Money for Sure w and on the no-borrowing incentive b, and thus we sometimes write $\tilde{\mu}(w, b)$ in the derivations. When $\tilde{\mu}$ has one argument, we continue to mean $\tilde{\mu}(b)$.

E.1 Proof of Proposition 1

We prove the following more general result:

Proposition (generalization of Prop 1) *Suppose that $\tilde{C}(x, \eta)$ is convex in x for all η. Then $\frac{\beta}{\tilde{\beta}} \geq \frac{l+p+b^\dagger}{l+p}$. Under the assumptions that (i) $\frac{k(l+p,\theta,\eta)-k(p,\theta,\eta)}{\tilde{C}(x_1,\eta)-\tilde{C}(x_2,\eta)} \perp \tilde{C}'(x, \eta) \forall \eta, x_1, x_2$, (ii) terms of order $(l + p)^3 E[\tilde{C}'''(x, \eta)/\tilde{C}'(x, \eta)]$ are negligible and (iii) $\tilde{\mu}$ is locally linear in b, we have the statement of Proposition 1.*

Proof. Define

$$G(b) := \beta E[\tilde{C}(l + p, \eta) - \tilde{C}(0, \eta)] - \tilde{\beta} E[\tilde{C}(l + p, \eta) - \tilde{C}(-b, \eta)]. \tag{17}$$

and note that $G(b^\dagger) = 0$ by definition. The convexity of $\tilde{C}$ implies that $\tilde{C}(0, \eta) \geq \frac{b^\dagger}{l+p+b^\dagger} \tilde{C}(l + p, \eta) + \frac{l+p}{l+p+b^\dagger} \tilde{C}(-b^\dagger, \eta)$ and thus

$$\begin{aligned} \frac{\beta}{\tilde{\beta}} &= \frac{E[\tilde{C}(l + p, \eta) - \tilde{C}(-b^\dagger, \eta)]}{E[\tilde{C}(l + p, \eta) - \tilde{C}(0, \eta)]} \\ &\geq \frac{E[\tilde{C}(l + p, \eta) - \tilde{C}(-b^\dagger, \eta)]}{\frac{l+p}{l+p+b^\dagger} E[\tilde{C}(l + p, \eta) - \tilde{C}(-b^\dagger, \eta)]} \\ &= \frac{l + p + b^\dagger}{l + p} \end{aligned}$$

Under the stronger assumptions, we have that up to negligible high-order terms,

$$\begin{aligned} G(b) &\approx \beta E\left[\tilde{C}'(0,\eta)(l+p)+\frac{\tilde{C}''(0,\eta)}{2}(l+p)^2\right] \\ &\quad - \tilde{\beta}E\left[(l+p+b)\tilde{C}'(0,\eta)+\frac{\tilde{C}''(0,\eta)}{2}(l+p-b)(l+p+b)\right] \\ &= \beta E\tilde{C}'\cdot(l+p)(1+\alpha/2\cdot(l+p)) - \tilde{\beta}(l+p+b)E\tilde{C}'\cdot[1+\alpha/2(l+p-b)] \\ &= \beta E\tilde{C}'\cdot(l+p)(1+\rho/2) - \tilde{\beta}(l+p+b)E\tilde{C}'\cdot(1+\rho/2-\alpha b/2). \end{aligned} \tag{18}$$

Setting $G(b^{\dagger}) = 0$ and dividing through by $E\tilde{C}'$, we have

$$\beta(l+p)(1+\rho/2) = \tilde{\beta}(l+p+b^{\dagger})\cdot(1+\rho/2-\alpha b^{\dagger}/2), \tag{19}$$

and thus

$$\frac{\beta}{\tilde{\beta}} \approx \frac{1+\rho/2-\alpha b^{\dagger}/2}{1+\rho/2}\frac{l+p+b^{\dagger}}{l+p}. \tag{20}$$

Finally, note that $b^{\dagger}$ is also the solution to $\tilde{\mu}(0,b^{\dagger}) = \mu(0,0)$. Thus $\tilde{\mu}(0,0) + b^{\dagger}\tilde{\mu}'_b(0,0) = \mu(0,0) + O((b^{\dagger})^2\tilde{\mu}''_b)$, and so

$$\begin{aligned} b^{\dagger} &\approx \frac{\mu(0,0)-\tilde{\mu}(0,0)}{-\tilde{\mu}'_b(0,0)} && (21) \\ &\approx \frac{-\gamma}{\tilde{\Delta}}\left(\mu(0,0)-\tilde{\mu}(0,0)\right) && (22) \end{aligned}$$

□

E.2 Proof of Proposition 2

We assume, in the more general case with η, that $\frac{k(l+p,\theta,\eta)-k(p,\theta,\eta)}{\tilde{C}(x_1,\eta)-\tilde{C}(x_2,\eta)} \perp \tilde{C}'(x,\eta)\forall\eta, x_1, x_2$. We define γ to be the size of the no-borrowing incentive used in the epxeriment.

Proof. In period $t-1$, the borrower believes that she will repay if

$$k(l+p,\theta,\eta) - k(p,\theta,\eta) \leq \tilde{\beta}[\tilde{C}(l+p-w,\eta) - \tilde{C}(-w-b,\eta)], \tag{23}$$

where w is money for sure and b is the size of the no-borrowing incentive. The assumption that $\frac{k(l+p,\theta,\eta)-k(p,\theta,\eta)}{\tilde{C}(x_1,\eta)-\tilde{C}(x_2,\eta)} \perp \tilde{C}'(x,\eta)\forall\eta, x_1, x_2$ implies that $\tilde{\mu}$ is not a function of η. If $\tilde{C}'$ is a function of η then the condition implies that $\frac{k(l+p,\theta,\eta)-k(p,\theta,\eta)}{\tilde{C}(x_1,\eta)-\tilde{C}(x_2,\eta)}$ must be constant in η for the assumption to be satisfied, and thus that the perceived probability of reborrowing is constant in η. If $\tilde{C}'$ is constant in η then η only affects k. In this case, η is an idiosyncratic liquidity shock like θ, and there is no loss in generality in simply allowing k be a function of θ only.

Thus, perceived reborrowing probability is a function of θ only, and we can thus define a cutoff $\theta^\dagger$, invariant in η, such that Equation (23) holds with equality at $\theta = \theta^\dagger$. This cutoff satisfies

$$\frac{d\theta^\dagger}{db} = \tilde{\beta}\frac{\tilde{C}'(x-b,\eta)}{k'_\theta(l+p,\theta^\dagger,\eta) - k'_\theta(p,\theta^\dagger,\eta)} \tag{24}$$

$$\frac{d\theta^\dagger}{dw} = -\tilde{\beta}\frac{\tilde{C}'(l+p-w,\eta) - \tilde{C}'(-w-b,\eta)}{k'_\theta(l+p,\theta^\dagger,\eta) - k'_\theta(p,\theta^\dagger,\eta)}. \tag{25}$$

Now

$$-\frac{\tilde{\mu}'_w}{\tilde{\mu}'_b} = \frac{-\frac{d\theta^\dagger}{dw}}{\frac{d\theta^\dagger}{db}} = \frac{\tilde{C}'(l+p-w,\eta)}{\tilde{C}'(-w-b,\eta)} - 1. \tag{26}$$

and

$$\begin{aligned}\frac{\tilde{C}'(l+p-w,\eta)}{\tilde{C}'(-w-b),\eta)} - 1 &= \frac{\tilde{C}'(l+p-w,\eta) - \tilde{C}'(-w-b,\eta)}{\tilde{C}'(-w-b)} \\ &= \frac{(l+p+b)\tilde{C}''(-w-b,\eta)}{\tilde{C}'(-w-b,\eta)} + O((l+p+b)^2\frac{\tilde{C}'''(-w-b,\eta))}{\tilde{C}'(-w-b,\eta)} \\ &= (l+p+b)\alpha(w,b,\eta) + O((l+p+b)^2\tilde{C}'''(-w-b,\eta)/\tilde{C}'(-w-b,\eta)).\end{aligned} \tag{27}$$

Thus $\rho(w,b,\eta) = -\frac{\tilde{\mu}'_w}{\tilde{\mu}'_b} + O((l+p+b)^2\tilde{C}'''/\tilde{C}')$, and thus does not vary with η up to negligible higher order terms. Similarly, $\alpha = \rho/(l+p+b)$ and thus also does not vary with η. We thus write $\rho(w,b)$ and $\alpha(w,b)$. Moreover,

$$\frac{d}{dw}\frac{\tilde{\mu}'_w}{\tilde{\mu}'_b} = \frac{\tilde{\mu}''_{ww}\tilde{\mu}'_b - \tilde{\mu}''_{wb}\tilde{\mu}'_w}{(\tilde{\mu}'_b)^2}, \tag{28}$$

and thus terms of order $\frac{d}{dw}\frac{\tilde{\mu}'_w}{\tilde{\mu}'_b}$ are negligible, which by Equation (27) implies that $\frac{d}{dw}\alpha$ and $\frac{d}{dw}\rho$ are negligible.

Now let

$$V(w,b) := -E\left[\int [k(l+p,\theta,\eta) + \tilde{C}(-w-b)]dF + \int_{\theta>\theta^\dagger} [k(p,\theta) + \tilde{C}(l+p-w)]dF\right]$$

denote the self $t-1$'s expected utility costs as a function of w and b. Our strategy is to characterize V as a function of w and b using second-order approximations of $\tilde{C}$, and to use those to quantify what value of w has the same impact on V as a change in b of size γ.

Ignoring higher-order negligible terms, we have

$$
\begin{aligned}
\frac{dV(0,b)}{db}(0,b,\eta_{t-1}) &= E\left[(1-\tilde{\mu})\tilde{C}'(-b,\eta) - (1-\tilde{\beta})(\tilde{C}(l+p,\eta)-\tilde{C}(-b,\eta))\tilde{\mu}_b'\right] \\
&= E\tilde{C}'(-b,\eta)\left[1-\tilde{\mu}-(1-\tilde{\beta})\frac{\tilde{C}(l+p,\eta)-\tilde{C}(-b,\eta)}{\tilde{C}'(a-b,\eta)}\tilde{\mu}_b'\right] \\
&= E\tilde{C}'(-b,\eta)\left[1-\tilde{\mu}-(1-\tilde{\beta})\frac{(l+p+b)\tilde{C}'+(l+p+b)^2/2\tilde{C}''}{\tilde{C}'}\tilde{\mu}_b'\right] \\
&= E\tilde{C}'(-b,\eta)\left[1-\tilde{\mu}-(1-\tilde{\beta})(l+p+b)\tilde{\mu}_b' - (1-\tilde{\beta})(l+p+b)^2\alpha(0,b)/2\tilde{\mu}_b'\right] \\
&= E\tilde{C}'(-b,\eta)\times \\
&\left[1-\tilde{\mu}-(1-\tilde{\beta})(l+p+b)\tilde{\mu}_b' + (1-\tilde{\beta})\frac{l+p+b}{2}\tilde{\mu}_w'\right]. \qquad (29)
\end{aligned}
$$

Differentiating again, and ignoring negligible terms, yields

$$
\begin{aligned}
\frac{d^2V(0,b)}{db^2} &= E\tilde{C}'(-b,\eta)\left[-\tilde{\mu}_b' - (1-\tilde{\beta})\tilde{\mu}_b' + (1-\tilde{\beta})\frac{1}{2}\tilde{\mu}_w'\right] \\
&- E\tilde{C}''(-b,\eta)\left[1-\tilde{\mu}-(1-\tilde{\beta})(l+p+b)\tilde{\mu}_b' + (1-\tilde{\beta})\frac{l+p+b}{2}\tilde{\mu}_w'\right], \qquad (30)
\end{aligned}
$$

which also implies that

$$
\frac{d^3V(0,b)}{db^3} = -2E\tilde{C}''(-b,\eta)\left[-\tilde{\mu}_b' - (1-\tilde{\beta})\tilde{\mu}_b' + (1-\tilde{\beta})/2\tilde{\mu}_w'\right] \qquad (31)
$$

and that fourth and higher derivatives of V are negligible. Thus, $V(0,\gamma,\eta) - V(0,0,\eta)$, divided through by $E\tilde{C}''(-b,\eta)$, is given by

$$
\begin{aligned}
&\left(V_b'|_{(0,0)}\gamma + V_b''|_{(0,0)}\gamma^2/2 + V_b'''|_{(0,0)}\gamma^3/6\right)/E\tilde{C}''(-b,\eta) \\
&= \gamma\left[1-\tilde{\mu}-(1-\tilde{\beta})(l+p+\gamma)\tilde{\mu}_b' + (1-\tilde{\beta})\frac{l+p+\gamma}{2}\tilde{\mu}_w'\right] \\
&+ \frac{\gamma^2}{2}\left[-\tilde{\mu}_b' - (1-\tilde{\beta})\tilde{\mu}_b' + (1-\tilde{\beta})/2\tilde{\mu}_w'\right] \\
&- \frac{\gamma^2}{2}\alpha\left[1-\tilde{\mu}-(1-\tilde{\beta})(l+p+\gamma)\tilde{\mu}_b' + (1-\tilde{\beta})\frac{l+p+\gamma}{2}\tilde{\mu}_w'\right] \\
&- \frac{\gamma^3}{3}\alpha\left[-\tilde{\mu}_b' - (1-\tilde{\beta})\tilde{\mu}_b' + (1-\tilde{\beta})/2\tilde{\mu}_w'\right] \\
&\approx (1-\alpha\gamma/2)\left[\gamma(1-\tilde{\mu}(0)+\tilde{\Delta}) + (1-\tilde{\beta})\tilde{\Delta}(l+p+\frac{\gamma}{2})(1+\rho/2)\right]. \qquad (32)
\end{aligned}
$$

In the derivations above, we use that (i) $\tilde{\mu}_b' = -\tilde{\Delta}/\gamma$ up to negligible higher-order terms, (ii) the property $E\tilde{C}''(-b,\eta)/E\tilde{C}'(-b,\eta) = \alpha$, which is obtained form the fact that α does not vary with η

as shown above, and the (iii) the additional approximation the term

$$\frac{\gamma^3}{12}\alpha\left[-\tilde{\mu}_b' - (1-\tilde{\beta})\tilde{\mu}_b' + (1-\tilde{\beta})/2\tilde{\mu}_w'\right] = -\frac{\gamma^2}{12}\alpha\left[\tilde{\Delta}\cdot(1+(1-\tilde{\beta})+(1-\tilde{\beta})\rho/2\right]$$

is numerically negligible. The additional rounding in (iii) slightly simplifies the presentation of the formula, but a more precise statement of the proposition can be obtained by adding this term back in.

Similarly, we compute how V changes with respect to w.

$$\begin{aligned}
\frac{dV(w,0)}{dw} &= E\left[\tilde{\mu}\tilde{C}'(l+p-w,\eta) + (1-\tilde{\mu})\tilde{C}'(-w,\eta) - (1-\tilde{\beta})(\tilde{C}(l+p-w,\eta)+\tilde{C}(-w,\eta))\tilde{\mu}_w'\right] \\
&= E\left[\tilde{\mu}\tilde{C}'(-w,\eta) + (1-\tilde{\mu})\tilde{C}'(-w,\eta) + \tilde{\mu}(l+p)\tilde{C}''(-w,\eta)\right] \\
&- E\left[(1-\tilde{\beta})(\tilde{C}(l+p-w,\eta)-\tilde{C}(-w,\eta))\tilde{\mu}_w'\right] \\
&= E\tilde{C}'(-w,\eta)\left[1+\tilde{\mu}(l+p) - (1-\tilde{\beta})\frac{\tilde{C}(l+p-w,\eta)-\tilde{C}(-w,\eta)}{\tilde{C}'(-w,\eta)}\tilde{\mu}_w'\right] \\
&= E\tilde{C}'(-w,\eta)\left[1+\tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})\frac{(l+p)\tilde{C}'+(l+p)^2/2\tilde{C}''}{\tilde{C}'(l+p-w)}\tilde{\mu}_w'\right] \\
&= E\tilde{C}'(-w,\eta)\left[1+\tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})(l+p)\tilde{\mu}_a' + (1-\tilde{\beta})(l+b)^2\alpha/2\tilde{\mu}_w'\right] \\
&= E\tilde{C}'(-w,\eta)\left[1+\tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1+\frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right].
\end{aligned} \tag{33}$$

Differentiating again yields

$$\begin{aligned}
\frac{d^2V}{dw^2} &= E\tilde{C}'(-w,\eta)\cdot\tilde{\mu}_w'(l+p)\alpha \\
&- E\tilde{C}''(-w,\eta)\cdot\left[1+\tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1+\frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right]
\end{aligned} \tag{34}$$

and

$$\frac{d^3V}{dw^3} = -2E\tilde{C}''(-w,\eta)\cdot\tilde{\mu}_w'(l+p)\alpha, \tag{35}$$

with fourth and higher derivatives of V negligible. Thus the impact of sure money w is equal to

$$
\begin{aligned}
&\left(V_w|_{(0,0)} w + V_w''|_{(0,0)} w^2/2 + V_w'''|_{(0,0)} w^3/6\right) / E\tilde{C}'(-w,\eta) \\
&= w\left[1 + \tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1 + \frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right] \\
&+ \frac{w^2}{2}\tilde{\mu}_w'(l+p)\alpha \\
&- \frac{w^2}{2}\alpha\left[1 + \tilde{\mu}(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1 + \frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right] \\
&- \frac{w^3}{3}\alpha^2\tilde{\mu}_w'(l+p) \\
&\approx w\left[1 + (\tilde{\mu} + (w/2)\tilde{\mu}_w')(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1 + \frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right] \\
&- \frac{w^2}{2}\alpha\left[1 + (\tilde{\mu} + (w/2)\tilde{\mu}_w')(l+p)\alpha - (1-\tilde{\beta})(l+p)\left(1 + \frac{l+p}{2}\alpha\right)\tilde{\mu}_w'\right] \\
&= w(1-\alpha w/2)\left[1 + (\tilde{\mu} + w/2\tilde{\mu}_w')\rho - (1-\tilde{\beta})(l+p)\left(1+\rho/2\right)\tilde{\mu}_w'\right] \\
&= w(1-\alpha w/2)\left[1 + \rho(\tilde{\mu} - (w\rho/2)\tilde{\mu}_b') + (1-\tilde{\beta})(l+p)\left(1+\rho/2\right)\rho\tilde{\mu}_b'\right] \\
&= (1-\alpha w/2)\times \\
&\left[w\left((1 + \rho\tilde{\mu} + \rho^2\frac{w}{2\gamma}\tilde{\Delta}\right) + (1-\tilde{\beta})(l+p)\left(1+\rho/2\right)\rho\frac{w}{\gamma}\tilde{\Delta}(\gamma)\right]
\end{aligned}
\tag{36}
$$

In the derivations above, we use that (i) $\tilde{\mu}_b' = -\tilde{\Delta}/\gamma$ up to negligible higher-order terms, (ii) the property $E\tilde{C}''(-b,\eta)/E\tilde{C}'(-b,\eta) = \alpha$, which is obtained form the fact that α does not vary with η as shown above, and the (iii) the additional approximation the term

$$\frac{w^3}{12}\alpha^2\tilde{\mu}_w' \cdot (l+p) = -\frac{w^3}{12}\alpha^2\rho\tilde{\mu}_b' \cdot (l+p) = \frac{w^3}{12}\alpha\rho^2\tilde{\Delta}/\gamma$$

is numerically negligible. The additional rounding in (iii) slightly simplifies the presentation of the formula, but a more precise statement of the proposition can be obtained by adding this term back in.

This implies that for non-marginal changes,

$$1-\tilde{\beta} = \frac{w(1-\alpha w/2)\left((1 + \rho\tilde{\mu} + \rho^2\frac{w}{2\gamma}\tilde{\Delta}(\gamma)\right) - \gamma(1-\alpha\gamma/2)(1-\bar{\tilde{\mu}}(\gamma))}{(1-\alpha\gamma/2)\tilde{\Delta}(\gamma)(l+p+\frac{\gamma}{2})(1+\rho/2) + (1-\alpha w/2)(l+p)\left(1+\rho/2\right)\rho\frac{w}{\gamma}\tilde{\Delta}(\gamma)}. \tag{37}$$

□

E.3 Deriving curvature from valuation of the \$100 coin flip

We have already shown that $\tilde{C}''(x,\eta)/\tilde{C}'(x,\eta)$ does not vary with η under the assumption that $k(l+p,\eta)/k(p,\eta) \perp \tilde{C}'(x,\eta)$ for all η. We now make the further assumption that $\alpha(x)$ is approximately constant in x. Thus, $\tilde{C}(x,\eta)$ can be approximated by $\lambda(\eta)\exp(\alpha \cdot x) + \lambda_0(\eta)$, for some functions λ and λ_0 of η, where $\eta \perp \theta$. This assumption is necessary to ensure that if c is the certainty equivalent for a Flip-a-Coin of size b, then c has the same effect on repayment behavior as does the Flip-a-Coin. Without an approximately constant level of absolute risk aversion, the certainty equivalent and the gamble can have different effects on period 1 behavior. Note that for low values of α, $\tilde{C}(x,\eta) = \alpha\tilde{C}'(0,\eta) + \frac{\alpha^2}{2}\tilde{C}''(0,\eta) + O(\alpha^3)$, and thus the quadratic approximation assumed in the body of the paper holds.

Formally, consider a period $t = 1$ certainty equivalent $c_1(\theta)$ for a Flip-a-Coin of size γ, as a function of θ. When θ is below the repayment threshold $\theta^\dagger$, we have

$$\exp(-\alpha c_1(\theta)) = \frac{1}{2}\exp(-\alpha b) + \frac{1}{2}\exp(0). \tag{38}$$

When θ is above the repayment threshold $\theta^\dagger$, we have

$$\exp(\alpha(l + p - c_1(\theta))) = \frac{1}{2}\exp(\alpha(l + p - b)) + \frac{1}{2}\exp(0),$$

which reduces to (38). Thus c^1 is independent of θ, and thus the period-0 certainty equivalent c simply satisfies

$$\exp(-\alpha c) = \frac{1}{2}\exp(-\alpha b) + \frac{1}{2}\exp(0). \tag{39}$$

In our empirical implementation, we estimate α assuming homogeneous risk preferences, and thus that variation in certainty equivalents c_i reflects mean-zero noise. This implies that the estimate $\hat{\alpha}$ satisfies

$$\exp(-\hat{\alpha}E[c_i]) = \frac{1}{2}\exp(-\hat{\alpha}b) + \frac{1}{2}\exp(0). \tag{40}$$

Inserting the empirical values, we have

$$\exp(-\hat{\alpha}42) = \frac{1}{2}\exp(-\hat{\alpha}100) + \frac{1}{2}\exp(0), \tag{41}$$

which is satisfied at $\hat{\alpha} \approx 0.0064$.

E.4 Empirical implementation

In theory, Equations (6) and (8) hold for each individual borrower, and individual survey responses could imply individual-specific β and $\tilde{\beta}$. In practice, any survey responses are noisy, so we observe $\tilde{\mu}$, $w(100)$, and risk aversion with measurement error. Furthermore, since Equations (6) and (8) involve squaring some of these survey response variables, we have non-classical measurement error

that cannot be addressed by simply taking expectations.

To address this, we define groups of observations indexed by g and calculate group-level averages of the empirical objects in Equations (6) and (8), including $b_g^\dagger = \frac{100}{\tilde{\mu}_g(0)-\tilde{\mu}_g(\gamma)}\left(\mu_g(0) - \tilde{\mu}_g(0)\right)$. For our primary estimates, we use the five groups defined by quintiles of loan size l. For shorthand in this appendix, we let w_i denote $w(100)_i$, person i's valuation of the \$100 no-borrowing incentive.

In Appendix E.4.1, we show how we can substitute the group average variables into Equations (6) and (8), take expectations over observations, and re-arrange, giving the following estimating equations:

$$\widehat{\left(\frac{\beta}{\tilde{\beta}}\right)} = \frac{\sum_i \left(l_g + p_g + b_g^\dagger\right)\left(1 + \frac{\rho_g}{2} - \frac{\alpha_g}{2} b_g^\dagger\right)}{\sum_i \left(l_g + p_g\right)\left(1 + \frac{\rho_g}{2}\right)} \tag{42}$$

and

$$\hat{\tilde{\beta}} = 1 - \frac{\sum_i \left\{ w_g \cdot \left(1 + \rho_g\left(\tilde{\mu}_g(0) + \frac{w_g\rho_g}{\gamma}\frac{\tilde{\Delta}_g}{2}\right)\right)\left(1 - \frac{\alpha_g w_g}{2}\right) - \gamma \cdot \left(1 - \tilde{\mu}_g(0) + \frac{\tilde{\Delta}_g}{2}\right)\left(1 - \frac{\alpha_g \gamma}{2}\right)\right\}}{\sum_i \left(1 + \frac{\rho_g}{2}\right)\tilde{\Delta}_g \left\{\left(l_g + p_g + \frac{\gamma}{2}\right)\left(1 - \frac{\alpha_g\gamma}{2}\right) + \left(l_g + p_g\right)\frac{w_g\rho_g}{\gamma}\left(1 - \frac{\alpha_g w_g}{2}\right)\right\}}. \tag{43}$$

If $\beta/\tilde{\beta}$ and $\tilde{\beta}$ are homogeneous across borrowers, Equations (42) and (43) deliver unbiased estimates as long as survey response error is mean-zero, so the expectations of w_g, $\tilde{\mu}_g$, μ_g, and $b_g^\dagger$ equal the true group means. If $\beta/\tilde{\beta}$ and $\tilde{\beta}$ are heterogeneous, Appendix E.4.1 lays out a set of assumptions under which Equations (42) and (43) are unbiased. One key assumption is that terms of order $E[(1-\tilde{\beta}_i)^2|g]$ are negligible. This is reasonable because if we think that no borrowers have $\tilde{\beta} > 1$, not many borrowers can plausibly have $\tilde{\beta}_i \ll 1$ given that our empirical estimate is that the average $\tilde{\beta}_i$ is not far from one. A second key assumption is that $E[\tilde{\beta}_i|g]$ does not vary across g. This is reasonable if we think that perceived time inconsistency is unrelated to loan size. Other papers that estimate present focus models (e.g. Laibson et al. 2015; Skiba and Tobacman 2018) assume homogeneity or comparable orthogonality conditions, although Bai et al. (2018) estimate a distribution of unobserved heterogeneity.

Empirically, we do not find any evidence that loan size is related to the present focus parameters once we control for experience effects. When adding loan size as covariate to the regression in column 1 of Table 2, the coefficient on loan size is not statistically significant at conventional levels ($p = 0.14$). When adding loan size as a covariate to the regression in column 5 of Table 2, the coefficient on loan size is also not statistically significant at conventional levels ($p = 0.20$). The effects of loan size are also insignificant when we add it as a covariate to the regressions in columns 3 and 7 of Table 2.

If $\beta/\tilde{\beta}$ and $\tilde{\beta}$ are homogeneous or $\beta_i/\tilde{\beta}_i$ is orthogonal to $\tilde{\beta}_i$ (i.e., people who think they are more versus less present focused misperceive their true present focus by the same proportion on average), then the sample average β is simply

$$\hat{\beta} = \widehat{\left(\frac{\beta}{\tilde{\beta}}\right)} \cdot \hat{\tilde{\beta}}. \tag{44}$$

Appendix E.4.1 shows that this estimator is also unbiased if β_i is orthogonal to a naivete statistic $\frac{1-\tilde{\beta}_i}{1-\beta_i}$ introduced by Augenblick and Rabin (2019), which reflects the degree to which people think their present focus is closer to 1 than it actually is.

E.4.1 Derivation of estimating equations

In this appendix, we show formally how our estimating equations can be derived from the formulas in Propositions 1 and 2, delivering the mean $\tilde{\beta}$ and β across heterogeneous borrowers. Define x_g as $E\left[x|g\right]$, the expectation of variable x in subsample g. We impose the following assumptions.

Assumption 1. *Any measurement error in l_i, p_i, w_i, $\tilde{\mu}_i$, μ_i, α_i, and ρ_i is mean-zero.*

Assumption 2. *l_i, p_i, α_i (and thus ρ_i) are homogeneous within a subsample g.*

Assumption 3. *Terms of order $E[(1-\tilde{\beta}_i)^2|g]$ and $E[(1-\beta_i/\tilde{\beta}_i)^2|g]$ are negligible.*

Assumption 4. *$Cov\left[E\left[\frac{\beta_i}{\tilde{\beta}_i}|g\right], (l_g+p_g)\left(1+\frac{\rho_g}{2}\right)\right] = 0.$*

Assumption 5. *$b_i^\dagger \perp \mu_i(b)$ for all b.*

Assumption 6. *$\tilde{\mu}_i$ is locally linear in b.*

Assumption 7. *$E\left[\tilde{\beta}_i|g\right]$ does not vary with g.*

Assumption 8. *Either $\beta_i/\tilde{\beta}_i \perp \tilde{\beta}_i$ or $\beta_i \perp \frac{\tilde{\beta}_i-\beta_i}{1-\beta_i}$.*

Assumption 3 is increasingly violated at lower values of $\tilde{\beta}$. However, if $\tilde{\beta}=1$ is an upper bound, our estimate of average $\tilde{\beta}\approx 0.76$ at $\alpha = 0.0064$ limits how small $\tilde{\beta}$ might plausibly be. For example, for a population with fairly extreme heterogeneity, with $\tilde{\beta}=1$ and $\tilde{\beta}=0.6$ each with probability 0.5, then $E[(1-\tilde{\beta}_i)^2] = 0.5\cdot(0.4)^2 = 0.08$.

Assumption 8 is that naivete is independent of perceived present focus. That is, we expect that people who perceive high $\tilde{\beta}$ misperceive β by the same proportion as people who perceive lower $\tilde{\beta}$.

Estimating $\beta/\tilde{\beta}$. To derive the estimating equation for sophistication, we begin with Equation (6). Imposing Assumptions 1 and 2 and re-arranging gives

$$\frac{\beta_i}{\tilde{\beta}_i} = \frac{\left(l_g+p_g+b_i^\dagger\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}b_i^\dagger\right)}{\left(l_g+p_g\right)\left(1+\frac{\rho_g}{2}\right)}. \tag{45}$$

Now note that from the proof of Proposition 1, we have that

$$0 \geq -b_i^\dagger \geq (l+p)(1-\beta_i/\tilde{\beta}_i) \tag{46}$$

This implies that $O(Var[b_i^\dagger|g])$ is $O(E[(1-\beta_i/\tilde{\beta}_i)^2|g])$ and thus negligible under Assumption 3. Taking expectations over borrowers within a group g gives

$$\begin{aligned} E\left[\frac{\beta_i}{\tilde{\beta}_i}|g\right] &= \frac{E\left[\left(l_g+p_g+b_i^\dagger\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}b_i^\dagger\right)|g\right]}{(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)} \\ &= \frac{\left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right)-\frac{\alpha_g}{2}Var[b_i^\dagger|g]}{(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)} \\ &= \frac{\left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right)}{(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)}+O(E[(1-\beta_i/\tilde{\beta}_i)^2|g]). \end{aligned} \tag{47}$$

Rearranging, we have

$$\begin{aligned} E\left[\frac{\beta_i}{\tilde{\beta}_i}|g\right](l_g+p_g)\left(1+\frac{\rho_g}{2}\right) &= \left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right) \\ \Leftrightarrow E\left[E\left[\frac{\beta_i}{\tilde{\beta}_i}|g\right](l_g+p_g)\left(1+\frac{\rho_g}{2}\right)\right] &= E\left[\left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right)\right] \\ \Leftrightarrow E\left[\frac{\beta_i}{\tilde{\beta}_i}\right]E\left[(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)\right] &= E\left[\left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right)\right] \\ \Leftrightarrow E\left[\frac{\beta_i}{\tilde{\beta}_i}\right] &= \frac{E\left[\left(l_g+p_g+E[b_i^\dagger|g]\right)\left(1+\frac{\rho_g}{2}-\frac{\alpha_g}{2}E[b_i^\dagger|g]\right)\right]}{E\left[(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)\right]}, \end{aligned} \tag{48}$$

where the third line follows from Assumption 4.

Finally, note that $\gamma(\mu_i(0)-\tilde{\mu}_i(0)) = -b_i^\dagger(\tilde{\mu}_i(0)-\tilde{\mu}_i(\gamma))$. To a first-order approximation, $\mu_i(0)-\tilde{\mu}_i(0) = -\frac{(\tilde{\beta}_i-\beta_i)}{\tilde{\beta}_i}\tilde{\mu}_i'(0)$ and $\tilde{\mu}_i(0)-\tilde{\mu}_i(\gamma) = -\gamma\mu_i'(0)\frac{\tilde{\beta}_i}{\beta_i}$, and thus by Assumption 5,

$$\begin{aligned} E\left[\frac{\gamma(\mu_i(0)-\tilde{\mu}_i(0))}{(\tilde{\mu}_i(0)-\tilde{\mu}_i(\gamma))}|g\right]-\frac{E[\mu_i(0)-\tilde{\mu}_i(0))|g]}{E[(\tilde{\mu}_i(0)-\tilde{\mu}_i(\gamma))|g]} &= E\left[\frac{(\tilde{\beta}_i-\beta_i)}{\tilde{\beta}_i}\right]-\frac{E\left[\frac{(\tilde{\beta}_i-\beta_i)}{\beta_i}\mu_i'(0)\right]}{E\left[\frac{\tilde{\beta}_i}{\beta_i}\mu_i'(0)\right]} \\ &= E\left[-\frac{\beta_i}{\tilde{\beta}_i}|g\right]+\frac{1}{E\left[\frac{\tilde{\beta}_i}{\beta_i}|g\right]} \\ &= E\left[-\frac{\beta_i}{\tilde{\beta}_i}|g\right]+E\left[-\frac{\beta_i}{\tilde{\beta}_i}|g\right]+O(E[(1-\beta_i/\tilde{\beta}_i)^2|g]) \end{aligned}$$

which implies that $E[b_i^\dagger|g] = -\gamma\frac{E[\mu_i(0)-\tilde{\mu}_i(0))|g]}{E[(\tilde{\mu}_i(0)-\tilde{\mu}_i(\gamma))|g]} = b_g^\dagger$ up to negligible higher-order terms. Substituting that in gives

$$E\left[\frac{\beta_i}{\tilde{\beta}_i}\right] = \frac{E\left[\left(l_g+p_g-b_g^\dagger\right)\left(1+\frac{\rho_g}{2}+\frac{\alpha_g}{2}b_g^\dagger\right)\right]}{E\left[(l_g+p_g)\left(1+\frac{\rho_g}{2}\right)\right]}. \tag{49}$$

The empirical analogue is the estimating equation, Equation (42).

Estimating $\tilde{\beta}$. To derive the estimating equation for $\tilde{\beta}$, we begin with Equation (8). Imposing Assumptions 1 and 2, re-arranging, and taking expectations over borrowers within a group g gives

$$E\left[\left(1-\tilde{\beta}_i\right)\left(1+\frac{\rho_g}{2}\right)\tilde{\Delta}_i\left\{\left(l_g+p_g+\frac{\gamma}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right)+(l_g+p_g)\frac{w_i\rho_g}{\gamma}\left(1-\frac{\alpha_g w_i}{2}\right)\right\}|g\right]$$
$$=E\left[\left\{w_i\cdot\left(1+\rho_g\left(\tilde{\mu}_g(0)+\frac{w_i\rho_g}{\gamma}\frac{\tilde{\Delta}_i}{2}\right)\right)\left(1-\frac{\alpha_g w_i}{2}\right)-\gamma\cdot\left(1-\tilde{\mu}_i(0)+\frac{\tilde{\Delta}_i}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right)\right\}|g\right]. \tag{50}$$

Now note that $Var[w_i|g]$, $Cov[\tilde{\Delta}_i, w_i|g]$, $Cov[w_i, \tilde{\beta}_i|g]$, $Cov[\tilde{\beta}_i, \tilde{\Delta}_i|g]$, $Cov[\tilde{\beta}_i, \tilde{\mu}_i(0)|g]$ are $O(E(1-\tilde{\beta}_i)^2)$ because Assumptions 1, 2, and 5 imply that conditional on g, the only variation in w_i and $\tilde{\Delta}_i$ is through $\tilde{\beta}_i$. They are then negligible under Assumption 3. The above equation thus reduces to

$$E\left[1-\tilde{\beta}_i|g\right]\left(1+\frac{\rho_g}{2}\right)\tilde{\Delta}_g\left\{\left(l_g+p_g+\frac{\gamma}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right)+(l_g+p_g)\frac{w_g\rho_g}{\gamma}\left(1-\frac{\alpha_g w_g}{2}\right)\right\}$$
$$=w_g\cdot\left(1+\rho_g\left(\tilde{\mu}_g(0)+\frac{w_g\rho_g}{\gamma}\frac{\tilde{\Delta}_g}{2}\right)\right)\left(1-\frac{\alpha_g w_g}{2}\right)-\gamma\cdot\left(1-\tilde{\mu}_g(0)+\frac{\tilde{\Delta}_g}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right). \tag{51}$$

Taking the expectation over groups g and applying Assumption 7 then implies that

$$E\left[\tilde{\beta}_i\right]=1-\frac{E\left[\left\{w_g\cdot\left(1+\rho_g\left(\tilde{\mu}_g(0)+\frac{w_g\rho_g}{\gamma}\frac{\tilde{\Delta}_g}{2}\right)\right)\left(1-\frac{\alpha_g w_g}{2}\right)-\gamma\cdot\left(1-\tilde{\mu}_g(0)+\frac{\tilde{\Delta}_g}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right)\right\}\right]}{E\left[\left(1+\frac{\rho_g}{2}\right)\tilde{\Delta}_g\left\{\left(l_g+p_g+\frac{\gamma}{2}\right)\left(1-\frac{\alpha_g\gamma}{2}\right)+(l_g+p_g)\frac{w_g\rho_g}{\gamma}\left(1-\frac{\alpha_g w_g}{2}\right)\right\}\right]}. \tag{52}$$

The empirical analogue is the estimating equation, Equation (43).

Backing out β. Taking a Taylor expansion, we have

$$E\left[\frac{\beta_i}{\tilde{\beta}_i}\right]=\frac{E[\beta_i]}{E[\tilde{\beta}_i]}+O(Cov[\beta_i,\tilde{\beta}_i])+O(E[(1-\tilde{\beta}_i)^2])$$

Since assumption 3 guarantees that $O(E[(1-\tilde{\beta}_i)^2])$ is negligible, we need only show that $O(Cov[\beta_i, \tilde{\beta}_i])$ is negligible. Now if the first condition of assumption 8 holds, then $O(Cov[\beta_i, \tilde{\beta}_i]) = O(E[(1-\tilde{\beta}_i)^2])$. Consider then the second condition, and set $\nu_i \equiv \frac{\tilde{\beta}_i-\beta_i}{1-\beta_i}$. Then $\beta_i = \frac{\tilde{\beta}_i-\nu_i}{1-\nu_i}$, and since $\beta_i \perp \nu_i$, this implies that $O(Cov[\beta_i, \tilde{\beta}_i]) = O(E[(1-\tilde{\beta}_i)^2])$. Thus $E\left[\frac{\beta_i}{\tilde{\beta}_i}\right] = \frac{E[\beta_i]}{E[\tilde{\beta}_i]}$ up to negligible higher-order terms.

E.5 Other sources of misprediction

In Section 7, we interpreted misprediction $\mu(0) - \tilde{\mu}(0)$ through the lens of a present focus model. However, misprediction could also be driven by overoptimism about future income or expenditure needs, or inattention to changes in those variables (Browning and Tobacman 2015; Karlan et al. 2016; Gabaix 2017). Our estimates of $\tilde{\beta}$ do not depend on the magnitude or source of misprediction, but our estimates of β could be affected.

For example, consider a model in which borrowers perceive that period 1 repayment costs k will be factor $\kappa \leq 1$ as large as they actually are. Then the right-hand side of Equation (3) is multiplied by $1/\kappa$, and the right-hand side of Equation (42) delivers an estimate of $\kappa \cdot \beta/\tilde{\beta}$. Intuitively, naivete about present focus causes a borrower to think that the period 1 self will give immediate costs $\beta/\tilde{\beta}$ less weight than she does in reality, which is mathematically isomorphic to believing that period 1 costs will be κ smaller than they are in reality (Browning and Tobacman 2015).

If $\kappa \neq 1$, we cannot estimate β with Equation (44). However, if we assume that $\kappa \leq 1$ and $\beta/\tilde{\beta} \leq 1$—that is, that borrowers are not *under*optimistic and do not perceive themselves to be *future* focused—then we can bound β on $\left[\widehat{\left(\frac{\beta}{\tilde{\beta}}\right)} \cdot \hat{\tilde{\beta}}, \hat{\tilde{\beta}}\right]$ using the estimates from Equations (42) and (43). The lower bound is from the assumption that $\kappa = 1$, so all of misprediction is driven by naivete about present focus, while the upper bound is from the assumption that $\beta/\tilde{\beta} = 1$, so all of misprediction is driven by other factors.

While Section 6 showed that borrowers overestimated the effect the incentive would have on borrowing, this should not affect our parameter estimates. We do not use misprediction in the Incentive condition to identify $\beta/\tilde{\beta}$, and our estimate of $\tilde{\beta}$ and the above bounds on β are valid as long as respondents correctly reported their beliefs on average on the survey. This would be the case under either of two plausible microfoundations for misprediction of the incentive effect: if $\kappa < 1$ and borrowers thus underestimate the *variance* in repayment cost shocks, or if they forget about the incentive with some probability in period 1, but naively fail to foresee this in period 0.

E.6 Implications if participants misreported their beliefs

Our estimation strategy assumes that people correctly reported their beliefs (on average) on the survey. If people instead overstated their true beliefs $\tilde{\Delta}$ about the effect of the incentive, our parameter estimates can be interpreted as bounds. $\tilde{\beta}$ would be an upper bound (i.e. people perceive more present focus than we estimate), because if predicted behavior is less responsive to the incentive than people report, their internality reduction per unit of behavior change is higher than we estimate. Sophistication would be an upper bound (i.e. $\beta/\tilde{\beta}$ will be smaller than we estimate), because if predicted demand is less responsive than people report, a given amount of misprediction in the Control condition implies a larger difference in predicted versus actual marginal utility.

To explore possible magnitudes, we estimate $\beta/\tilde{\beta}$ and $\tilde{\beta}$ with alternative equations where we set $\tilde{\Delta}$ to half its reported amount, keeping $\alpha = 0.0064$. That is, we assume that people report that

the incentive will reduce their borrowing probability by twice as much as they actually believe. Under this assumption, $\tilde{\beta}$ drops substantially to 0.54, and $\beta/\tilde{\beta}$ decreases to 0.95. Thus, to estimate $\tilde{\beta}$, it is crucial to assume that people reported their beliefs without bias. However, the fact that this alternative assumption delivers a low $\tilde{\beta}$ that is out of line with estimates from other domains provides additional support for the assumption that people did correctly report their beliefs.

F Additional Results and Proofs for Section 8

F.1 Existence and uniqueness of equilibrium

We suppose that the period t cost of repaying an amount x is $k_t(x, \theta, \eta)$. As before, we assume that in each period $t \geq 1$, the borrower can either choose to repay p, $l + p$, or default. We also assume that for the infinite-horizon case ($T = \infty$), there is some finite T' after which $k_t(x, \theta, \eta)$ does not vary with t.

We divide the shocks to costs of repayment into two components: an i.i.d. component and a serially correlated component. We set $\omega_t = (\theta_t, \eta_t)$, where $\theta_t \sim F$ denotes the i.i.d. component and $\eta_t \sim G(\cdot|\eta_{t-1})$ denotes the serially correlated component. We let G_0 denote the distribution of η in period 1.

We make several regularity assumptions on the distribution θ and the cost of repayment k.

Assumption 9. *The distribution of θ has a smooth density function f with convex and compact support.*

Assumption 10. *$k(x, \theta, \eta)$ is twice differentiable in all three arguments.*

Assumption 11. *For all $x_2 > x_1$ and η, $k(x_2, \theta, \eta) - k(x_1, \theta, \eta)$ is increasing in θ, with $\lim_{\theta \to \infty} k(x_2, \theta, \eta) - k(x_1, \theta, \eta) = \infty$.*

Assumption 12. *For all x and η, $k(x, 0, \eta) = 0$, and for all x,θ,η, $k(0, \theta, \eta) = 0$.*

Assumption 13. *For all finite $x \geq 0$ and η, $\int_\theta k(x, \theta, \eta) dF(\theta) < \infty$.*

Assumption 14. *The distributions $G(\cdot|\eta)$ have common finite support, and $G(\eta|\eta) > 1/2$, $G(\eta'|\eta) > 0$ for $\eta' \in$ supp G.*

Let $\tilde{r}(l, \eta_t)$ denote the period $\tau < t$ perceived continuation value of starting off in period t with a loan of size l after experiencing a shock η_t in period t. This is different from $\tilde{C}(l, \eta_t)$, which is the period t self's perceived continuation value of starting period $t + 1$ in with debt l. The two are linked by the relationship $\tilde{C}(l, \eta_t) = \sum_{\eta'} \tilde{r}(l, \eta') G(\eta'|\eta_t)$. For the proofs in the appendix, however, it will be convenient to utilize $\tilde{r}$.

For our purposes, it is also useful to consider the fee $p < l$ as fixed, and the repayment rule to be that the borrower must pay either pay p or repay in full or default.

In period $t - 1$ the individual defaults if

$$\min\left(k_t(l + p, \theta_{t-1}, \eta_{t-1}), \beta\delta E[\tilde{r}(l, \theta, \eta)|\eta_{t-1}] + k_t(p, \theta_{t-1}, \eta_{t-1})\right) \geq \chi. \tag{53}$$

Conditional on not defaulting, the individual chooses to repay if

$$k_t(l+p,\theta_{t-1},\eta_{t-1}) \leq \beta\delta E[\tilde{r}(l,\theta,\eta)|\eta_{t-1}] + k_t(p,\theta_{t-1},\eta_{t-1}). \tag{54}$$

In periods $\tau < t-1$ the individual thinks he will choose to default if

$$\min\Big(k_t(l+p,\theta_{t-1},\eta_{t-1}), \tilde{\beta}\delta E[\tilde{r}(lp,\theta,\eta)|\eta_{t-1}] + k_t(p,\theta_{t-1},\eta_{t-1})\Big) \geq \chi, \tag{55}$$

and if he does not default, then he thinks he will repay in period t if

$$k_t(l+p,\theta_{t-1},\eta_{t-1}) \leq \tilde{\beta}\delta E[\tilde{r}(p,\theta,\eta)|\eta_{t-1}] + k_t(p,\theta_{t-1},\eta_{t-1}). \tag{56}$$

We begin considering the case with infinite horizon and time-invariant cost-of-repayment functions ($k_t \equiv k$ for all t)

Theorem 1. *Suppose that $T = \infty$ and $k_t \equiv k$ for all t. For each l, there exists a unique stationary equilibrium with a continuation value function $\tilde{C}(\eta)$ that is twice differentiable in l.*

Proof. Say there are J elements in the union of the supports of $G(\cdot|\eta)$, defined as $S_G = \bigcup_\eta \text{supp}G(\cdot|\eta)$, with elements enumerated $\eta_1,\ldots,\eta_J$. Then $\tilde{r}(\cdot)$ is a vector in $\mathbb{R}^J$, and we adopt the convention that $\tilde{r}(\eta_i)$ corresponds to the ith component of the vector.

For any function $h : S_G \to \mathbb{R}^J$, define $\bar{h}(\eta) = \sum_{\eta' \in S_G} h(\eta')G(\eta'|\eta)$. Define the operator B_i on functions $h : S_G \to \mathbb{R}^J$ as follows:

$$\begin{aligned} B_i(h) &= Pr(D(h,\eta_i))\chi \\ &+ \int_{\theta \leq c(h,\eta_i)} \mathbf{1}_{\theta \notin D(h,\eta_i)} k(l+p,\theta,\eta_i)dF + \int_{\theta \geq c(h,\eta_i)} \mathbf{1}_{\theta \notin D(h,\eta_i)} \left(\delta\bar{h}(\eta_i) + k(p,\theta,\eta_i)\right) dF, \end{aligned} \tag{57}$$

where $c(h,\eta)$ is the solution to

$$k(l+p,c,\eta) = \tilde{\beta}\delta\bar{h}(\eta) + k(p,c,\eta), \tag{58}$$

which is unique by Assumption 11, and

$$D(h,\eta_i) := \{\theta | \min\Big(k(l+p,\theta,\eta_i), \tilde{\beta}\delta\bar{h}(\eta_i) + k(p,\theta,\eta_i)\Big) \geq \chi\}. \tag{59}$$

Since $k(l+p,\theta,\eta_i), \tilde{\beta}\delta\bar{h}(\eta_i)+k(p,\theta,\eta_i)$ are both increasing in θ, $\min\Big(k(l+p,\theta,\eta_i), \tilde{\beta}\delta\bar{h}(\eta_i) + k(p,\theta,\eta_i)\Big)$ is increasing in θ, and thus there is a unique cutoff $d(h,\eta_i)$ such that $\theta \in D(h,\eta_i)$ iff $\theta \geq d(h,\eta_i)$.

By definition, the perceived continuation value $\tilde{r}(\cdot)$ must be a fixed point of the map $B(h) = (B_1(h),\ldots,B_J(h))$

Now observe that $B_i(h) > 0$ and that $B_i(h) < \chi/\tilde{\beta}$. Consequently, $B(h) \in [0,\chi/\tilde{\beta}]^J$ for all h. By Brouwer's fixed point theorem, continuity of B is thus sufficient to establish that B has a fixed point inside $[0,\chi/\tilde{\beta}]^J$.

Set $m^+(h,\eta_i) = \max(c(h,\eta_i), d(h,\eta_i))$ and $m^-(h,\eta_i) = \min(c(h,\eta_i), d(h,\eta_i))$. Clearly, c and d are both differentiable in h, and thus m^+ and m^- are continuous and almost everywhere differentiable in h. Now if $c > d$ then the borrower repays in full when $\theta < d$ and defaults when $\theta > d$. When $c < d$ the borrower repays in full when $\theta < c$, rolls over the loan when $\theta \in (c,d)$, and defaults when $\theta > d$. Therefore,

$$B_i(h) = \int_{\theta \geq m^+(h,\eta_i)} \chi f(\theta)d\theta + \int_{\theta \leq m^-(h,\eta_i)} k(l+p,\theta,\eta_i)f(\theta)d\theta \tag{60}$$

$$+ \int_{c(h,\eta_i) \leq \theta \leq m_i^+(h,\eta_i)} \left(\delta\bar{h}(\eta_i) + k(p,\theta,\eta_i)\right) f(\theta)d\theta, \tag{61}$$

where the integral in Equation (61) is equal to zero if $c(h,\eta_i) = d_i^+(h,\eta_i)$. Each integral above is continuous in h. Consequently, B is continuous in h, which establishes that it has a fixed point.

We now move on to consider uniqueness. First, note that c and d can be expressed as functions of $\bar{h}(\eta_i)$ only, and do not depend separately on $h(\eta_i)$ conditional on that mean. Consequently, $B_i(h)$ is a function of $\bar{h}(\eta_i)$ only. Our strategy is to show that $\frac{d}{d\bar{h}(\eta_i)}B_i(h) < 1$ at all points of differentiability. This will imply the result because if h and h' are both fixed points of B, so that $B_i(h) = h(\eta_i)$ and $B_i(h') = h'(\eta_i)$ $\forall i$, then we reach a contradiction as follows: Assume, without loss of generality, that $h(\eta_i) - h'(\eta_i) = \max_j |h(\eta_j) - h'(\eta_j)| > 0$ for some i. By assumption 14, this implies that $\bar{h}(\eta_i) \geq \bar{h}'(\eta_i)$. Since $\bar{h}(\eta_i) - \bar{h}'(\eta_i) \leq h(\eta_i) - h'(\eta_i)$ by construction, we obtain the contradiction that

$$\begin{aligned} h(\eta_i) - h'(\eta_i) &= B_i(h) - B_i(h') \\ &< \bar{h}(\eta_i) - \bar{h}'(\eta_i) \\ &\leq h(\eta_i) - h'(\eta_i). \end{aligned} \tag{62}$$

To show that $\frac{d}{d\bar{h}(\eta_i)}B_i(h) < 1$, we consider three cases in turn. First, $d < c$, second, $d > c$, and third d higher than the maximum of the support of θ (consumers never default at η_i). In all cases we use the fact that c is increasing in $\bar{h}(\eta_i)$ while d is decreasing in $\bar{h}(\eta_i)$, which follows from Assumption 11.

In the first case, $k(l+p,d),\eta_i) = \chi$, and thus

$$B_i(h) = \int_{\theta \leq d(h,\eta_i)} k(l+p,\theta,\eta_i)f(\theta)d\theta + \int_{\theta \geq d(h,\eta_i)} \chi f(\theta)d\theta \tag{63}$$

$$\begin{aligned} \frac{\partial}{\partial\bar{h}(\eta_i)}B_i &= (k(l+p,d,\eta_i) - \chi)\frac{\partial d}{d\bar{h}(\eta_i)}f(d) \\ &= 0. \end{aligned} \tag{64}$$

In the second case, $\tilde{\beta}\delta\bar{h}(\eta_i) + k(p,d,\eta) = \chi$, and thus

$$\begin{aligned}
B_i(h) &= \int_{\theta \le c(h,\eta_i)} k(l+p,\theta,\eta_i) f(\theta) d\theta \\
&\quad + \int_{c(h,\eta_i) \le \theta \le d(h,\eta_i)} \left(\delta \bar{h}(\eta_i) + k(p,\theta,\eta_i)\right) f(\theta) d\theta + \int_{\theta > d(h,\eta_i)} \chi f(\theta) d\theta, \qquad (65) \\
\frac{\partial}{\partial \bar{h}(\eta_i)} B_i &= \left(k(l+p,c,\eta_i) - \delta \bar{h}(\eta_i) - k(lp,\theta,\eta_i)\right) f(c) \frac{\partial c}{\partial \bar{h}(\eta_i)} \\
&\quad + \left(\delta \bar{h}(\eta_i) + k(p,d,\eta_i) - \chi\right) f(d) \frac{\partial d}{\partial \bar{h}(\eta_i)} + \int_{c(h,\eta_i) \le \theta \le d(h,\eta_i)} \delta f(\theta) d\theta \\
&= -(1-\tilde{\beta}) \delta \bar{h}(\eta_i) f(c) \frac{\partial c}{\partial \bar{h}(\eta_i)} + (1-\tilde{\beta}) \delta \bar{h}(\eta_i) f(d) \frac{\partial d}{\partial \bar{h}(\eta_i)} + \int_{c(h,\eta_i) \le \theta \le d(h,\eta_i)} \delta f(\theta) d\theta \\
&< \int_{c(h,\eta_i) \le \theta \le d(h,\eta_i)} \delta f(\theta) d\theta \le \delta. \qquad (66)
\end{aligned}$$

Note that in the limit of $d \to c$ in this second case, $\tilde{\beta}\delta\bar{h}(\eta) + k(p,c,\eta) = k(l+p,c,\eta) \to \chi$, and thus $\frac{\partial d}{\partial \bar{h}(\eta_i)} = \frac{\partial c}{\partial \bar{h}(\eta_i)}$. Thus B_i is differentiable in h at $c = d$, and $\frac{\partial}{\partial \bar{h}(\eta_i)} B_i \to 0$ in the limit of $|d - c| \to 0$

In the third case,

$$\begin{aligned}
B_i(h) &= \int_{\theta \le c(h,\eta_i)} k(l+p,\theta,\eta_i) f(\theta) d\theta + \int_{c(h,\eta_i) \le \theta)} \left(\delta \bar{h}(\eta_i) + k(p,\theta,\eta_i)\right) f(\theta) d\theta \qquad (67) \\
\frac{\partial}{\partial \bar{h}(\eta_i)} B_i &= \left(k(l+p,c,\eta_i) - \delta \bar{h}(\eta_i) - k(p,\theta,\eta_i)\right) f(c) \frac{\partial c}{\partial \bar{h}(\eta_i)} + \int_{c(h,\eta_i) \le \theta} \delta f(\theta) d\theta \\
&= \left(\tilde{\beta} \delta \bar{h}(\eta_i) - \delta \bar{h}(\eta_i)\right) f(c) \frac{\partial c}{\partial \bar{h}(\eta_i)} + \int_{c(h,\eta_i) \le \theta} \delta f(\theta) d\theta \\
&< \int_{c(h,\eta_i) \le \theta} \delta f(\theta) d\theta \le \delta \qquad (68)
\end{aligned}$$

It is clear that the derivative in the third case is the limit case in which d approaches the supremum of the support of θ. Thus B_i is differentiable in h when d is at the maximum of the support. Consequently, the steps above also show that B_i is everywhere differentiable in h. Finally, similar reasoning shows that B is also twice differentiable in h as well as twice differentiable in l. The implicit function theorem then implies that the unique fixed point of B must be twice differentiable in l. □

Extension to finite horizon and partial time-invariance

The result holds immediately for the case of finite horizon. Plainly, in period $t = T$ the continuation-value function is unique, and is twice differentiable because the density function is smooth. The continuation-value functions in periods $t < T$ are obtained by repeated application of the bellman operator B defined above. Because we have already shown that it is twice differentiable, the result follows immediately for finite horizon. Similarly, if $T = \infty$ and the cost functions are time-invariant

starting at $t = T'$, then $\tilde{r}_{T'}$ are unique and twice-differentiable (once we apply the stationarity requirement that the equilibrium is stationary when the cost functions are stationary), and $\tilde{r}_t$ for $t < T'$ can be obtained from $\tilde{r}_{T'}$ by repeated application of the bellman operator.

F.2 Continuity

For simplicity, we assume that $T = \infty$ and that the distribution of costs is time invariant. It is trivial to generalize to the slightly more general set up above.

Proposition 4. *A borrower's period 0 expected utility is continuous in $\beta, \tilde{\beta}_0, \tilde{\beta}_1$.*

Proof. Paralleling the perceived continuation value definition in the proof of Theorem 1, we define $r(l, \eta_i)$ to be the objectively expected continuation value of starting out period t with loan l if η_i is realized.

Let $c^*(\eta)$ be the value of c satisfying

$$k(l + p, c, \eta) = \beta\delta\bar{\tilde{r}}(\eta) + k(l, c, \eta), \tag{69}$$

which is unique by Assumption 11. (Recall that for any function $h : S_G \to \mathbb{R}^J$, we defined the "bar" notation to be $\bar{h}(\eta) = \sum_{\eta' \in S_G} h(\eta')G(\eta'|\eta)$.) Set $d^*(\eta)$ to be the unique value of d that satisfies

$$\min\left(k(l + p, d, \eta), \beta\delta\bar{\tilde{r}}(\eta) + k(p, d, \eta)\right) = \chi. \tag{70}$$

Set $m^{*+}(h, \eta_i) = \max(c^*(\eta_i), d^*(\eta_i))$ and $m^{*-}(h, \eta_i) = \min(c^*(\eta_i), d^*(\eta_i))$. Then the objective expectation of continuation value is the fixed point of $B^* = (B_1^*, \ldots B_M^*)$ given by

$$\begin{aligned} B_i^*(h) = &\int_{\theta \geq m^{*+}(\eta_i)} \chi f(\theta) d\theta + \int_{\theta \leq m^{*-}(\eta_i)} k(l + p, \theta, \eta_i) f(\theta) d\theta \\ &+ \int_{c^*(\eta_i) \leq \theta \leq m_i^{*+}(\eta_i)} \left(\delta\bar{h}(\eta_i) + k(p, \theta, \eta_i)\right) f(\theta) d\theta. \end{aligned} \tag{71}$$

Because $\bar{\tilde{r}}$ is uniquely determined, so are $c^*(\eta_i)$ and $d^*(\eta_i)$. Thus, the above is simply a system of M linear equations in M unknowns, which has a unique solution.

Plainly, d^* and c^* are continuous in β and $\bar{\tilde{r}}$. And since $\bar{\tilde{r}}$ is continuous in $\tilde{\beta}$, this implies that both are continuous in $\tilde{\beta}$ as well. Finally, $B_i^*(h)$ is clearly continuous in d^* and c^*, which implies the continuity result of the proposition.

□

F.3 Results with vanishing and maximal uncertainty

To parametrize the degree of uncertainty, we consider a family of distributions F_l and G_l, with G_λ^0 and $G(\cdot|\eta_j)$ all having a common support for all λ, and with $\theta \in [0, \bar{\theta}]$ and $\eta_j \in [0, \bar{\eta}]$. We suppose that the means $E_{F_\lambda}[\theta]$, $E_{G_\lambda^0}[\eta]$, and $E_{G_\lambda}[\eta|\eta_j]$ do not depend on λ. Defining $\sigma_F^2, \sigma_{G^0}^2, \sigma_{G(\cdot|\eta_j)}^2$ to be

the maximum variance of distributions with respective means $E_{F_\lambda}[\theta]$, $E_{G^0_\lambda}[\eta]$, and $E_{G_\lambda}[\eta|\eta_j]$ and supports within $[0,\bar{\theta}]$, $[0,\bar{\eta}]$, $[0,\bar{\eta}]$,[36] we assume that

1. $\lim_{\lambda\to 0} Var_{F_\lambda}[\theta] = 0, \lim_{\lambda\to 0}\max_{ij}(\eta_i - \eta_j) = 0$
2. $\lim_{\lambda\to\infty} Var_{F_\lambda}[\theta] = \sigma^\infty(\mu)$, $\lim_{\lambda\to\infty} Var_{G^0_\lambda}[\eta] = \sigma^2_{G^0}$, $\lim_{\lambda\to\infty} Var_{G_\lambda}[\eta|\eta_j] = \sigma^2_{G(\cdot|\eta_j)}$

We also make the normalization assumption that for all x, $k(x,\theta,\eta) = 0$ when $\theta = 0$. We consider the welfare and policy implications of bias under two extreme cases: (i) minimal uncertainty, represented by $\lambda \to 0$ and (ii) high uncertainty, represented by $\lambda \to \infty$. When studying (i), we focus on the interesting case in which $E[k(l + p(l), \theta)] < \chi$, so that it is not optimal to default immediately. In the statements below, we use $E[k(x), \omega)]$ to denote the expectation with respect to the period 1 distribution of $\omega = (\eta, \eta)$.

Proposition 5. *Define $\bar{\theta}$ as the upper bound of the support of $F(\theta)$. For $\bar{\theta}$ high enough,* $\lim_{\lambda\to\infty}\left(C_\lambda(l) - C^{TC}_\lambda\right) = 0$ *and* $\lim_{\lambda\to\infty}\left(\tilde{C}_\lambda(l) - C^{TC}_\lambda\right) = 0.$

Proposition 6. *Suppose that $E[k(l + p(l), \theta, \eta)] < \chi$, so that it is not optimal to default in period $t = 1$. If $\beta \geq \beta^* := \frac{E[k(l+p(l),\theta,\eta)]-E[k(p(l),\theta,\eta)]}{E[k(l+p(l),\theta,\eta)]}$, then the behavior of the present focused borrower approaches that of a time-consistent borrower as $\lambda \to 0$. Otherwise:*

1. If $\tilde{\beta}_1 > \beta$, then $\lim_{\lambda\to 0}\left(C_\lambda(l) - C^{TC}_\lambda(l)\right) = \infty$.

2. If $\tilde{\beta}_1 = \beta$, then $\lim_{\lambda\to 0} C_\lambda(l) = \frac{C^{TC}(l)-E[k(p(l),\theta,\eta)]}{\beta}$.

3. If $\tilde{\beta}_1 = \beta$, then $\lim_{\lambda\to 0}\frac{\tilde{C}_\lambda(l)}{C_\lambda(l)} = \beta\frac{\max\left(\left(C^{TC}(l)-E[k(p(l),\theta,\eta)]\right)/\tilde{\beta}_0, C^{TC}(l)\right)}{C^{TC}(l)-E[k(p(l),\theta,\eta)]} \in [\beta/\tilde{\beta}_0, 1]$. If $\tilde{\beta}_1 > \beta$, then $\lim_{\lambda\to 0}\frac{\tilde{C}_\lambda(l)}{C_\lambda(l)} = 0$.

F.4 Proof of Proposition 5

We begin with a series of lemmas.

Lemma 1. *The distributions F_λ and G_λ converge in distribution to distributions F_* and G_* such that F_* is Bernoulli on $[0,\bar{\theta}]$ and $G^0_*, G_*(\cdot|\eta_i)$ are Bernoulli on $[0,\bar{\eta}]$.*

Proof. The Bhatia-Davis inequality implies that given a constraint on the man and the support, the maximum variance is obtained by a Bernoulli distribution with all mass on the lower and upper bound of the support. For F_λ, this implies a variance equal to $\mu(\bar{\theta} - \mu)$, where $\mu = E_{F_\lambda}[\theta]$.

Now suppose, for the sake of contradiction, that the Lemma were not true for F_λ. Then there are some $\alpha > 0$ and $\epsilon > 0$ such that F_λ puts weight at least α on the probability that $\theta \in [\epsilon, \bar{\theta} - \epsilon]$

[36] By the Bhatia and Davis (2000) inequality, $\sigma^\infty(\mu)$ and $(\sigma^\infty(\mu_j))$ exist.

for all λ. Then

$$\begin{aligned} Var_{F_\lambda}[\theta] &= \int \theta^2 dF - \mu^2 \\ &\leq (1-\alpha)\int \bar{\theta}\theta dF_\lambda + \alpha \int (\bar{\theta}-\epsilon)\theta dF - \mu^2 \\ &= \bar{\theta}\mu - \mu^2 - \alpha\epsilon\mu. \end{aligned} \tag{72}$$

Consequently, the variance of F_λ is bounded away from the maximal possible variance, which contradicts the assumption that the variance of F_λ converges to the maximal possible variance.

By the same logic, G^0_λ and $G_\lambda(\cdot|\eta_i)$ converge to Bernoulli distributions as well. □

Lemma 2. *Let F^* and G^* be the distributions to which F_λ and G_λ converge. For $\bar{\theta}$ large enough, there is a unique stationary pure-strategy equilibrium under F^* and G^* that does not depend on β and $\tilde{\beta}$.*

Proof. We show that the for $\bar{\theta}$ large enough, the unique equilibrium is to repay when $\theta = 0$, and to delay or default when $\theta = \bar{\theta}$. Since the costs of repayment are zero for $\theta = 0$, it is clear that it is optimal to repay when $\theta = 0$. Now fixing $\bar{\eta}$, by Assumption 11 there is a $\bar{\theta}^\dagger$ high enough such that $k(l+p,\bar{\theta},\eta), k(p,\bar{\theta},\eta) > \chi$ for all $\eta \in [0,\bar{\eta}]$ and all $\bar{\theta} \geq \bar{\theta}^\dagger$. Thus for all such $\bar{\theta}$, the borrower defaults $\theta = \bar{\theta}$, irrespective of $\beta, \tilde{\beta}$. Thus, for $\bar{\theta}$ high enough, the borrower repays when $\theta = 0$ and defaults when $\theta = \bar{\theta}$, irrespective of the present focus parameters. □

Lemma 3. *For $\bar{\theta}$ large enough, the expected period 0 utility under F_λ, G_λ converges to expected period 0 utility under F_*, G_*.*

Proof. Let the common support of G_λ be $\eta_1 < \cdots < \eta_J$. Let $\tilde{r}_\lambda = (\tilde{r}_\lambda(\eta_i), \ldots \tilde{r}_\lambda(\eta_J))$ be the vector of perceived equilibrium continuation strategies for each F_λ, G_λ, and let r_* be the vector of continuation strategies corresponding to F_*, G_*. Note that by the assumption that G_λ have common support, Lemma 1 implies that $\eta_1 = 0$ and $\eta_J = \bar{\eta}$ and that $G^0_\lambda(\eta), G_\lambda(\eta|\eta_i) \to 0$ for $0 < \eta < \bar{\eta}$.

Now consider the best response correspondences B^λ, B^* with respect to F_λ,G_λ and F_*, G_*, respectively, defined in Equations (60,61). By the above, it is enough to show that $\tilde{r}_\lambda(0) \to r_*(0)$ and $\tilde{r}_\lambda(\bar{\eta}) \to r_*(\bar{\eta})$. To that end, note Lemma 1 implies that $G_\lambda(\cdot|\eta)$ converges to a Bernoulli distribution with support 0 and $\bar{\eta}$ and probability $\frac{\bar{\eta}}{E_{G_*}[\eta'|\eta]}$ of $\eta = \bar{\eta}$. Thus for any $h : \mathbb{R}^J \to \mathbb{R}^J$ and η

$$\sum_{\eta'} h(\eta')G_\lambda(\eta'|\eta) \to \left(1 - \frac{\bar{\eta}}{E_{G_*}[\eta'|\eta]}\right) h(0) + \frac{\bar{\eta}}{E_{G_*}[\eta'|\eta]} h(\bar{\eta}). \tag{73}$$

Consequently, if $k(l+p,\bar{\theta},\eta) - k(p,\bar{\theta},\eta) > \chi$ for all $\eta \in [0,\bar{\eta}]$, then by the reasoning of Lemma 2

and the fact that F_λ converges to a Bernoulli distribution with support $\{0, \bar{\theta}\}$,

$$B_j^\lambda(h) \to Pr_{F_*}(\theta = \bar{\theta})\frac{1}{\tilde{\beta}}\min\left(\tilde{\beta}\chi, \tilde{\beta}k(p, \bar{\theta}, \eta) + \left(1 - \frac{\bar{\eta}}{E_{G_*}[\eta|\eta_j]}\right)h(0) + \frac{\bar{\eta}}{E_{G_*}[\eta|\eta_j]}h(\bar{\eta})\right) \tag{74}$$

for all j and $h : \mathbb{R}^J \to \mathbb{R}^J$. Moreover, since $\tilde{r}_\lambda(\eta_i) \in [0, \chi/\tilde{\beta}]$ for all λ and η_i, we can restrict attention to $h \in [0, \chi/\tilde{\beta}]^J$, which allows us to strengthen the convergence in Equation (74) above to uniform convergence. This implies that $\lim_{\lambda\to 0}\tilde{r}_\lambda(0)$ and $\lim_{\lambda\to 0}\tilde{r}_\lambda(\bar{\eta})$ solve the system of linear equations

$$\lim_{\lambda\to 0}\tilde{r}_\lambda(0) = Pr_{F_*}(\theta = \bar{\theta})\frac{1}{\tilde{\beta}}\min\left(\tilde{\beta}\chi, \tilde{\beta}k(p, \bar{\theta}, 0) + \left(1 - \frac{\bar{\eta}}{E_{G_*}[\eta|0]}\right)\lim_{\lambda\to 0}\tilde{r}_\lambda(0) + \frac{\bar{\eta}}{E_{G_*}[\eta|0]}\lim_{\lambda\to 0}\tilde{r}_\lambda(\bar{\eta})\right) \tag{75}$$

$$\lim_{\lambda\to 0}\tilde{r}_\lambda(\bar{\eta}) = Pr_{F_*}(\theta = \bar{\theta})\frac{1}{\tilde{\beta}}\min\left(\tilde{\beta}\chi, \tilde{\beta}k(p, \bar{\theta}, \bar{\eta}) + \left(1 - \frac{\bar{\eta}}{E_{G_*}[\eta|\bar{\eta}]}\right)\lim_{\lambda\to 0}\tilde{r}_\lambda(0) + \frac{\bar{\eta}}{E_{G_*}[\eta|\bar{\eta}]}\lim_{\lambda\to 0}\tilde{r}_\lambda(\bar{\eta})\right) \tag{76}$$

but the system of equations above is precisely the system of equations that characterizes $r_*(0)$ and $r_*(\bar{\eta})$. $\square$

Proof of the main result

Proof. The result follows immediately from the three lemmas above, in particular Lemma 3. As shown in Lemma 3, the borrower either repays or defaults when the distribution of θ is Bernoulli with sufficiently wide support. The other lemmas simply establish convergence to this equilibrium.

We have that (i)$\lim_{\lambda\to 0}\tilde{r}_\lambda(0) = r_*(0)$, $\lim_{\lambda\to 0}\tilde{r}_\lambda(\bar{\eta}) = r_*(\bar{\eta})$; (ii) G_λ^0 and $G_\lambda(\cdot|\eta_j)$ converge to G_*^0 and $G_*(\cdot|\eta_j)$ and (iii) the uniform convergence condition of Equation (74), together with Lemma 3, imply that

$$\tilde{r}_\lambda(\eta) \to Pr_{F_*}(\theta = \bar{\theta})\frac{1}{\tilde{\beta}}\min\left(\tilde{\beta}\chi, \tilde{\beta}k(p, \bar{\theta}, \eta) + \left(1 - \frac{\bar{\eta}}{E_{G_*}[\eta|\eta]}\right)r_*(0) + \frac{\bar{\eta}}{E_{G_*}[\eta|\eta]}r_*(\bar{\eta})\right) \tag{77}$$

and thus that $\tilde{r}_\lambda(\eta)$ are all bounded away from zero. Now fixing $\bar{\eta}$, by Assumption 11 there is a $\bar{\theta}^\dagger$ high enough such that $k(l + p, \bar{\theta}, \eta), k(p, \bar{\theta}, \eta) > \chi$ for all $\eta \in [0, \bar{\eta}]$ and all $\bar{\theta} \geq \bar{\theta}^\dagger$. Under this condition, the individual does indeed repay for small enough θ, and defaults for θ sufficiently close to $\bar{\theta}$. $\square$

Extension to misprediction of costs

The arguments above extend verbatim to the case in which borrowers are sophisticated about their present focus but think that future costs are κ as high as they are.

F.5 Proof of Proposition 6

Proof. or shorthand, let $\bar{k}(x)$ denote the expectation of the period 1 costs of repayment, and let $\bar{k}(x, \eta_i)$ denote the expectation conditional on η_i. Let $\tilde{r}_\lambda(\eta_i)$ denote the expected cost, given a realization of η_i, with respect to the distributions F_l and G_l. Construct $\bar{\tilde{r}}_\lambda(\eta_i) := \sum_j \tilde{r}_\lambda(\eta_j)G(\eta_j|\eta_i)$. □

Lemma 4. $\lim_{\lambda\to 0} \max_{ij} |\bar{r}(\eta_i) - \bar{r}(\eta_j)| = 0$.

Proof. Note that as $\lambda \to 0$, there is no option value of delaying, and thus the time-consistent individual repays immediately. Any delays are suboptimal. Consequently, $\lim_{\lambda\to 0} \tilde{r}_\lambda(\eta_i) \geq \bar{k}(l+p)$ and $\lim_{\lambda\to 0} \bar{\tilde{r}}_\lambda(\eta_i) \geq \bar{k}(l+p)$ for all i.

Now consider first the case in which $\tilde{\beta}_1 \geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$. In this case

$$\begin{aligned}\lim_{\lambda\to 0}\left(k(p) + \tilde{\beta}_1 \bar{\tilde{r}}_\lambda(\eta_i)\right) &\geq k(p) + \tilde{\beta}_1 \bar{k}(l+p) \\ &\geq \bar{k}(l+p), \end{aligned} \tag{78}$$

and thus the borrower repays immediately. Consequently, $\lim_{\lambda\to 0} \bar{\tilde{r}}_i(l) = \bar{k}(l+p)$ for all i.

Next, consider the case in which $\tilde{\beta}_1 < \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$. We break up the proof into three cases.

Case 1. Suppose, toward a contradiction, that there exist $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that $\max_i \left(\bar{k}(p) + \tilde{\beta}_1 \bar{\tilde{r}}_\lambda(\eta_i)\right) > \bar{k}(l+p) + \epsilon_1$ and $\min_i \left(\bar{k}(p) + \tilde{\beta}_1 \bar{\tilde{r}}(\eta_i)\right) < \bar{k}(l+p) - \epsilon_1$for all λ. Then there exists $\bar{\lambda} > 0$ such that $\max_i \left(\bar{k}(p, \eta_i) + \tilde{\beta}_1 \bar{\tilde{r}}_\lambda(\eta_i)\right) > \bar{k}(l+p, \eta_i) + \epsilon_1$ and $\min_i \left(\bar{k}(p, \eta_i) + \tilde{\beta}_1 \bar{\tilde{r}}_\lambda(\eta_i)\right) < \bar{k}(l+p, \eta_i) - \epsilon_1$ for all $\lambda \leq \bar{\lambda}$. Consequently, $Pr\left(\max_i \left(k(p, \theta, \eta_i) + \tilde{\beta}_1 \bar{\tilde{r}}_\lambda(\eta_i)\right) > \bar{k}(l+p, \theta, \eta_i) + \epsilon_1\right) \to 1$, meaning that the probability that the borrower thinks he chooses to repay that period approaches 1. Thus if $\bar{i}(\lambda)$ is the index that maximizes $\bar{\tilde{r}}_\lambda(\eta_i)$, then $\tilde{r}_\lambda(\eta_{\bar{i}}) \to \bar{k}(l+p)$. Similarly, if $\underline{i}(\lambda)$ is the index that minimizes $\bar{\tilde{r}}_i(\eta_i)$ then in this case the probability that the borrower thinks he chooses to repay approaches 0, and $\tilde{r}_\lambda(\eta_{\underline{i}}) \to \bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) + \bar{k}(p)$.

Now since by assumption $G(\eta_i|\eta_i) > 1/2$ for all i, and since $\lim_{\lambda\to 0} \tilde{r}_\lambda(\eta_i) \geq \bar{k}(l+p) \forall i$, we have that

$$\begin{aligned}\lim_{\lambda\to 0} \bar{\tilde{r}}_\lambda(\eta_{\bar{i}}) &\leq \frac{1}{2}\bar{k}(l+p) + \frac{1}{2}\lim_{\lambda\to 0} \max_i \tilde{r}_\lambda(i) \\ &\leq \frac{1}{2}\bar{k}(l+p) + \frac{1}{2}\lim_{\lambda\to 0} \left(k(p) + \bar{\tilde{r}}_\lambda(\eta_i)\right) \\ \Leftrightarrow \lim_{\lambda\to 0} \bar{\tilde{r}}_\lambda(\eta_{\bar{i}}) &\leq \bar{k}(p) + \bar{k}(l+p), \end{aligned} \tag{79}$$

and

$$\begin{aligned}\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) &\geq \frac{1}{2}\bar{k}(l+p)+\frac{1}{2}\lim_{\lambda\to 0}\tilde{r}(\eta_{\underline{i}})\\ &= \frac{1}{2}\bar{k}(l+p)+\frac{1}{2}\lim_{\lambda\to 0}\left(\bar{\tilde{r}}_\lambda(\eta_{\underline{i}})+\bar{k}(p)\right)\\ \Leftrightarrow \lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) &\geq \bar{k}(p)+\bar{k}(l+p), \end{aligned}\tag{80}$$

which implies that $\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) \geq \lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}})$—a contradiction.

Case 2. Suppose that there exists $\epsilon > 0$ such that $\max_i\left(\bar{k}(p)+\tilde{\beta}_1\bar{\tilde{r}}_\lambda(\eta_i)\right) > \bar{k}(l+p)+\epsilon$ for all λ. Now if there exists ϵ such that $\bar{k}(p)+\tilde{\beta}_1\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_i) > \bar{k}(l+p)+\epsilon$ for all i, then by the reasoning in Case 1, the borrower never delays repayment in the limit, and thus $\bar{\tilde{r}}_\lambda(\eta_i)\to\bar{k}(l+p)$ for all i, which is impossible when $\tilde{\beta}_1 < \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$. Thus by Case 1,

$$\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) = \frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1} > \bar{k}(l+p) = \lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}),\tag{81}$$

which again generates a contradiction.

Case 3. Suppose, toward a contradiction, that there exists $\epsilon > 0$ such that $\min_i\left(\bar{k}(p)+\tilde{\beta}_1\bar{\tilde{r}}_\lambda(\eta_i)\right) < \bar{k}(l+p)-\epsilon$ for all λ. Now if there exists ϵ such that $\bar{k}(p)+\tilde{\beta}_1\bar{\tilde{r}}(\eta_i) < \bar{k}(l+p)-\epsilon$ for all i, then by the reasoning in Case 1, the borrower always delays repayment in the limit, and thus $\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_i)=\infty$ for all i. Thus by Case 1, $\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\bar{i}}) = \frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1}$, and therefore for μ denoting the probability of transition from$\underline{i}$ to a state in which the agent does not delay with probability 1:

$$\begin{aligned}\lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) &\geq \mu\frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1}+(1-\mu)\lim_{\lambda\to 0}\tilde{r}(\eta_{\underline{i}})\\ &= \mu\frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1}+(1-\mu)\lim_{\lambda\to 0}\left(\bar{\tilde{r}}_\lambda(\eta_{\underline{i}})+\bar{k}(p)\right)\\ \Leftrightarrow \lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\underline{i}}) &\geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1}+\frac{(1-\mu)}{\mu}\bar{k}(p)\\ &> \frac{\bar{k}(l+p)-\bar{k}(p)}{\tilde{\beta}_1}\\ &= \lim_{\lambda\to 0}\bar{\tilde{r}}_\lambda(\eta_{\bar{i}}),\end{aligned}\tag{82}$$

which generates a contradiction. □

The above lemma implies that the pure strategies and payoffs of the setting with diminishing uncertainty converge to the stationary mixed strategy equilibrium of a game with no uncertainty, in which the cost of delay is $\bar{k}(p)$ and the cost of paying immediately is $\bar{k}(l+p)$. For the proof of the proposition, we therefore consider the stationary mixed strategy equilibria of the game with no uncertainty.

Proof of the main result

Proof. We first characterize the perceived equilibrium in terms of $\tilde{\beta}$ in the case of no uncertainty. If $\tilde{\beta} \geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ then the agent perceives himself to repay immediately. To see this, note that if $\tilde{C}$ is the expected continuation cost, then it cannot be lower than $\bar{k}(l+p)$ since the optimal strategy for no uncertainty is to repay immediately. Thus,

$$\begin{aligned} k(p) + \tilde{\beta}_1 \tilde{C} &\geq k(p) + \tilde{\beta}_1 \bar{k}(l+p) \\ &\geq \bar{k}(l+p), \end{aligned} \tag{83}$$

Assume now that $\tilde{\beta} < \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ and let $\tilde{\mu}$ be the perceived probability of repaying next period. For the mixed strategy to be feasible, the agent must be indifferent between continuing or not in the next period, and thus the continuation cost $\tilde{C}$ must satisfy $\bar{k}(p) + \tilde{\beta}\tilde{C} = \bar{k}(l+p)$, or

$$\tilde{C} = \frac{\bar{k}(l+p) - \bar{k}(p)}{\tilde{\beta}}. \tag{84}$$

To solve for $\tilde{\mu}$, observe also that

$$\tilde{C} = \tilde{\mu}\bar{k}(l+p) + (1-\tilde{\mu})(\bar{k}(p) + \tilde{C}). \tag{85}$$

Solving Equations (84) and (85) yields

$$\tilde{\mu} = \frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{\bar{k}(p)}{\bar{k}(l+p) - \bar{k}(p)}. \tag{86}$$

Now if $\beta \geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ then the agent does indeed repay immediately.

If $\beta < \tilde{\beta}_1$ then the agent never repays since the perceived continuation cost $\tilde{C}_1$ in periods $t \geq 1$ satisfies equation (84) with $\tilde{\beta}_1$ in place of $\tilde{\beta}$, and thus $\bar{k}(p) + \beta\tilde{C}_1 < \bar{k}(l+p)$. Consequently, the agent simply accumulates infinite costs from continually paying the fee p.

If $\beta = \tilde{\beta}_1$ then the continuation cost is given by $C = \frac{\bar{k}(l+p)-\bar{k}(p)}{\beta}$

Parts 2 and 3 of the proposition follows simply from Equations (84) and (86), noting that $C^{TC} \to \bar{k}(l+p)$. □

Extension to misprediction of costs

Suppose instead that borrowers perceive future costs to be $\kappa \leq 1$ as high as they are. Lemma 4 holds verbatim. Moreover, $\lim_{\lambda \to 0} \tilde{\bar{r}}(\eta_i) \geq \kappa \bar{k}(l+p)$ for all i so if $\beta/\kappa \geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ then

$$\begin{aligned} \lim_{\lambda \to 0} \left(k(p) + \tilde{\beta}_1 \tilde{\bar{r}}(\eta) \right) &\geq k(p) + \tilde{\beta}_1 \bar{k}(l+p) \\ &\geq \bar{k}(l+p), \end{aligned} \tag{87}$$

and the borrower perceives himself to repay immediately.

Assume now that $\beta/\kappa < \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ and let $\tilde{\mu}$ be the perceived probability of repaying next period. For the mixed strategy to be feasible, the agent must be indifferent between continuing or not in the next period, and thus the continuation cost $\tilde{C}$ must satisfy $\kappa\bar{k}(p) + \beta\tilde{C} = \kappa\bar{k}(l+p)$, or

$$\tilde{C} = \frac{\bar{k}(l+p) - \bar{k}(p)}{\beta/\kappa}. \tag{88}$$

Now if $\beta \geq \frac{\bar{k}(l+p)-\bar{k}(p)}{\bar{k}(l+p)}$ then the agent does indeed repay immediately.

If $\kappa < 1$ then the agent never repays since the perceived continuation cost $\tilde{C}_1$ in periods $t \geq 1$ satisfies Equation (88) with $\tilde{\beta}_1$ in place of $\tilde{\beta}$, and thus $\bar{k}(p) + \beta\tilde{C} < \bar{k}(l+p)$. Consequently, the agent simply accumulates infinite costs from continually paying the fee p.

If $\kappa = 1$ then the continuation cost is given by $C = \frac{\bar{k}(l+p)-\bar{k}(p)}{\beta}$. Parts 2 and 3 of the proposition follows simply from Equation (88), noting that $C^{TC} \to \bar{k}(l+p)$.

F.6 Proof of Proposition 3

Proof. Note that under the unconditional invariance assumption, for any function $u(\cdot)$, we have that $\sum_{\eta,\eta_i} u(\eta)G(\eta|\eta_i)G_0(\eta_i) = \sum u(\eta)G_0(\eta)$. We use this throughout.

We first solve for $C^S_\beta(\eta)$: the period t continuation cost function when a state η is realized in period $t \geq 1$. We suppress the loan size l as an argument for simplicity. Partition the set Θ into the sets D, RB, and RP where D is the set of all θ for which the borrower defaults, RB is the set for which the borrower re-borrows, RP is the set of all θ for which the borrower repays in full. Define

$$\begin{aligned} r^*(\eta_i) := \min_{D,RP,RB} \Bigg\{ &\int_{\theta\in D} \chi f(\theta)d\theta + \int_{\theta\in RB} \left(C^S_\beta(\eta_i) + k(p,\theta,\eta_i)\right) f(\theta)d\theta \\ &+ \int_{\theta\in RP} k(l+p,\theta,\eta_i)f(\theta)d\theta \Bigg\} \end{aligned} \tag{89}$$

That is, r^* is the minimum expected cost to a time-consistent borrower from the period 1 perspective, given a realization η_i at the beginning of period 1, but given the continuation value function $C^S_\beta(\eta_i)$ that corresponds to a present-focused borrower. Let $D^*_\eta, RP^*_\eta, RB^*_\eta$ denote the sets corresponding to this cost-minimizing strategy.

As before, define $r(\eta_i)$ as the actual expected period t cost of a borrower who realizes η_i in period t. Let D_η, RB_η, and RP_η be the sets corresponding to the actual strategy of the present-focused borrower. Define $\mu(\eta_i) = Pr(\theta \in RB_\eta)$ and assume that $\mu(\eta_i)$ has a weakly negative covariance with $C^S_\beta(\eta_i)$; that is, the borrower is more likely to reborrow when the continuation cost of doing so is lower. This assumption is automatically satisfied when $\mu(\eta_i)$ is constant, and thus when there is only a single realization of η_i; i.e., it is satisfied for "standard" optimal stopping problems with a time-invariant distribution of payoff shocks. Define $\mu = \sum_\eta \sum_{\eta_i} \mu(\eta)G(\eta|\eta_i)G_0(\eta_i)$

as the (unconditional) probability of reborrowing.

Note that $\theta \in RB_\eta$ if and only if $k(p,\theta,\eta) + \beta C^S_\beta(\eta) \le \min(k(l+p,\theta,\eta),\chi)$, or equivalently if and only if

$$k(p,\theta,\eta) + C^S_\beta(\eta) \le \min(k(l+p,\theta,\eta),\chi) + (1-\beta)C^S_\beta(\eta). \tag{90}$$

Thus, $RB^*_\eta \subset RB_\eta$. Moreover, note that for $\theta \notin RB_\eta$, the time-consistent and present-focused borrowers behave identically, as they both choose between the immediate costs of either reborrowing or repaying. Thus, their behavior differs only for $\theta \in RB_\eta \setminus RB^*_\eta$. For these values, behavior differs because

$$\min(k(l+p,\theta,\eta),\chi) \le k(p,\theta,\eta) + C^S_\beta(\eta) \le \min(k(l+p,\theta,\eta),\chi) + (1-\beta)C^S_\beta(\eta)$$

and thus relative to a time-consistent borrower with the same continuation value function, the size of the "internality" that the present-focused borrower imposes on herself cannot be more than $(1-\beta)C^S_\beta(\eta_i)$ in this case. Thus

$$r(\eta_i) \le r^*(\eta_i) + \mu(\eta_i)(1-\beta)C^S_\beta(\eta_i), \tag{91}$$

To bound the period 0 expected cost function C^S_β, we sum the above equation over all realizations of η_i, weighted by prior G_0:

$$\begin{aligned}
C^S_\beta &\le \sum_{\eta_i} r^*(\eta_i)G_0(\eta_i) + \sum_{\eta_i} \mu(\eta_i)(1-\beta)C^S_\beta(\eta_i)G_0(\eta_i) \\
&\le \sum_{\eta_i} r^*(\eta_i)G_0(\eta_i) + (1-\beta)\sum_{\eta_i} \mu(\eta_i)G_0(\eta_i)\sum_{\eta_i} C^S_\beta(\eta_i) \\
&= \sum_{\eta_i} r^*(\eta_i)G_0(\eta_i) + (1-\beta)\sum_{\eta_i} \mu(\eta_i)G_0(\eta_i)\sum_{\eta_i}\sum_{\eta} r(\eta)G(\eta|\eta_i)G_0(\eta_i) \\
&= \sum_{\eta_i} r^*(\eta_i)G_0(\eta_i) + (1-\beta)\mu\sum_{\eta} r(\eta_i)G_0(\eta_i) \\
&= \sum_{\eta_i} r^*(\eta_i)G_0(\eta_i) + (1-\beta)\mu C^S_\beta
\end{aligned} \tag{92}$$

To complete the proof for sophisticates, note that $\sum r^*(\eta)G_0(\eta)$ cannot be lower than C^{TC}; else, the time-consistent borrower could choose a better strategy. Thus,

$$C^S_\beta \le C^{TC} + \mu(1-\beta)C^S_\beta. \tag{93}$$

Rearranging gives the first result.

For partial naifs we again have $RB^*_\eta \subset RB_\eta$. It also continues to hold that if the borrower does not reborrow, then her choice of whether to default or repay in full corresponds to that of a

time-consistent borrower. However, Equation (90) is modified to

$$k(p,\theta,\eta)+C^{S}_{\tilde{\beta}}(\eta)\leq \min(k(l+p,\theta,\eta),\chi)+(1-\beta)C^{S}_{\tilde{\beta}}(\eta). \tag{94}$$

Adding $C^{PN}_{\beta,\tilde{\beta}}(\eta)-C^{S}_{\tilde{\beta}}(\eta)$ to both sides yields

$$k(p,\theta,\eta)+C^{PN}_{\beta,\tilde{\beta}}(\eta)\leq \min(k(l+p,\theta,\eta),\chi)+C^{PN}_{\beta,\tilde{\beta}}(\eta)-\beta C^{S}_{\tilde{\beta}}(\eta). \tag{95}$$

Proceeding as before, define

$$\begin{aligned} r^*(\eta_i) := \min_{D,RP,RB}\int_{\theta\in D}\chi f(\theta)d\theta+\int_{\theta\in RB}\left(C^{PN}_{\beta,\tilde{\beta}}(\eta_i)+k(p,\theta,\eta_i)\right)f(\theta)d\theta \\ +\int_{\theta\in RP}k(l+p,\theta,\eta_i)f(\theta)d\theta \end{aligned} \tag{96}$$

from which it follows that for partial naifs,

$$r(\eta_i)\leq r^*(\eta_i)+\mu(\eta_i)\left(C^{PN}_{\beta,\tilde{\beta}}(\eta_i)-\beta C^{S}_{\tilde{\beta}}(\eta_i)\right), \tag{97}$$

To complete the proof in analog to the proof for sophisticates, make the additional assumption that $\mu(\eta_i)$ has a weakly positive covariance with $C^{S}_{\tilde{\beta}}(\eta)-C^{PN}_{\beta,\tilde{\beta}}(\eta)$; that is, the states in which the borrower is most likely to reborrow are states where present focus, holding beliefs $\tilde{\beta}$ constant, leads the borrower to want to reborrow the most. This ensures that $\mu(\eta_i)$ has a weakly negative covariance with $C^{PN}_{\beta,\tilde{\beta}}(\eta_i)-\beta C^{S}_{\tilde{\beta}}(\eta_i)$.

As before, summing over the realizations of η_i and using the time-invariance of the unconditional distribution of η implies that

$$C^{PN}_{\beta,\tilde{\beta}}\leq C^{TC}+\mu C^{PN}_{\beta,\tilde{\beta}}-\mu\beta C^{S}_{\tilde{\beta}}. \tag{98}$$

Rearranging Equation (98) gives the second result in the proposition. □

Extension to misprediction of costs

By identical logic, let $C^{PN}_{\beta,\kappa}$ denote the continuation value function of an agent who perceive future costs to be only κ as high as they are, and let $C^{S}_{\beta,\kappa}(\eta)$ denote the the continuation value function that would result if the borrower was in fact right. Then

$$C^{PN}_{\beta,\kappa}\leq C^{TC}+\mu C^{PN}_{\beta,\kappa}(\eta)-\mu\beta C^{S}_{\beta,\kappa}, \tag{99}$$

and thus

$$C^{PN}_{\beta,\kappa}\leq \frac{C^{TC}}{1-\mu}-\frac{\mu}{1-\mu}\beta C^{S}_{\beta,\kappa}. \tag{100}$$

F.7 Additional calibration results

F.7.1 Conditions on marginal benefits and costs of first dollar borrowed

The surplus from borrowing will be positive when $l^* < 2l^\dagger$. To see this, first note that $C'(l^\dagger) = C'(0) + l^\dagger C''$ and $u'(l^\dagger) = u'(0) + l^\dagger u'' u'(0) = u'(l^\dagger) - l^\dagger u''(l^\dagger)$ and thus

$$\begin{aligned} u'(0) &= u'(l^\dagger) - l^\dagger u'' \\ &= C'(l^\dagger) - l^\dagger u'' \\ &= l^\dagger (C'' - u'') + C'(0) \end{aligned} \tag{101}$$

Borrower welfare at the optimal $l^\dagger$ is then given by

$$G := \frac{(u'(0) - C'(0))l^\dagger}{2} = \frac{C'' - u''}{2}(l^\dagger)^2. \tag{102}$$

If borrowers instead choose l^* to solve $u'(l) = \kappa C'(l)$—where either $\kappa = \beta/\tilde{\beta}_0$ or $k = \beta$, as in the body of the paper—then

$$L = \frac{1}{2}\Delta^2(C'' - u'').$$

Consequently, borrower welfare is positive if $l^\dagger > \Delta = l^* - l^\dagger$.

Now

$$l^\dagger = \frac{u'(0) - C'(0)}{(C'' - u'')} \tag{103}$$

and

$$l^* = \frac{u'(0) - \kappa C'(0)}{(\kappa C'' - u'')}. \tag{104}$$

Now $2l^\dagger > l^*$ if and only if

$$\begin{aligned} &2\frac{u'(0) - C'(0)}{(C'' - u'')} > \frac{u'(0) - \kappa C'(0)}{(\kappa C'' - u'')} \\ \Leftrightarrow &(u'(0) - \kappa C'(0))(C'' - u'') < 2(\kappa C'' - u'')(u'(0) - C'(0)) \\ \Leftrightarrow &\kappa C'(0)C'' - (2 - \kappa)u''C'(0) < (2\kappa - 1)C''u'(0) - u'(0)u'' \\ \Leftrightarrow &C'(0)\left[kC'' - (2 - \kappa)u''(0)\right] < u'(0)\left[(2\kappa - 1)C'' - u''\right] \\ \Leftrightarrow &\frac{u'(0)}{C'(0)} > \frac{\kappa C'' - (2 - \kappa)u''}{(2\kappa - 1)C'' - u''} \end{aligned} \tag{105}$$

Now since C'' and $-u''$ are both positive, the term $\frac{kC''-(2-\kappa)u''}{(2\kappa-1)C''-u''}$ is increasing in C''/u'' when $\frac{\kappa}{2\kappa-1} > (2 - \kappa)$ and $(2\kappa - 1) > 0$. This inequality holds for $\kappa \in (1/2, 1]$, as it is equivalent

to

$$
\begin{aligned}
&\kappa > (2\kappa-1)(2-\kappa)\\
\Leftrightarrow &0 > -2\kappa^2+4\kappa-2\\
\Leftrightarrow &0 > -2(\kappa-1)^2
\end{aligned} \tag{106}
$$

Inequality (105) thus holds if it holds in the limit case $C''/u'' \to \infty$, which reduces to $u'(0)/C'(0) > \kappa/(2\kappa-1)$ or, equivalently,

$$u'(0)/\tilde{C}'(0) > 1/(2\kappa-1). \tag{107}$$

For example, at $\kappa = 0.9$, $G > L$ if $u'(0)/\tilde{C}'(0) > 1/(1.8-1) = 1.25$. This is a worst-case bound that applies to cases in which $u''/C'' \approx 0$ (marginal costs increase much faster than marginal benefits), which is tightened to $u'(0) > C'(0)/\kappa$ when $u'' = C''$, and to $u'(0) > C'(0)(2-\kappa)$ when $C'' = 0$.

F.7.2 Calibrations with linear demand and marginal cost

To further explore how likely condition (105) is to hold, consider a population of potential borrowers with linear marginal benefits and costs given by $\theta_B u'(l)$ and $\theta_C C'(l)$, where (θ_B, θ_C) follow an arbitrary joint distribution. Suppose that $-u'(0)/u''(0) = C'(0)/C''(0) \equiv \alpha$. In reality, u' likely declines faster than C', as borrowers have a particular liquidity shock that they need to address, and the marginal benefits of borrowing an amount greater than the liquidity need are relatively small. Thus, the illustrative assumption that $-u'(0)/u''(0) = C'(0)/C''(0)$ is likely conservative, as the bounds for guaranteeing that $G > L$ become less demanding with more curvature in either u' or C'.

Then for $\theta := \theta_B/\theta_C \cdot u'(0)/C'(0)$,

$$
\begin{aligned}
l^*(\theta) &= \frac{\theta_B u'(0) - \kappa\theta_C C'(0)}{(\kappa\theta_C C'' - \theta_B u'')}\\
&= \frac{1}{\alpha}\frac{\theta-\kappa}{\theta+\kappa}
\end{aligned} \tag{108}
$$

The condition that $2l^\dagger > l^*$ is equivalent to

$$
\begin{aligned}
&2\frac{\theta_B u'(0) - \theta_C C'(0)}{(\theta_C C'' - \theta_B u'')} > \frac{\theta_B u'(0) - \kappa\theta_C C'(0)}{(\kappa\theta_C C'' - \theta_B u'')}\\
\Leftrightarrow &\frac{2}{\alpha}\frac{\theta-1}{\theta+1} > \frac{1}{\alpha}\frac{\theta-\kappa}{\theta+\kappa}.
\end{aligned} \tag{109}
$$

For $\kappa = 0.9$, equation (109) holds when $\theta > 1.1$. What does this translate to for $l^*(\theta)$? Setting $\alpha = 0.002$, which is more three times smaller than our empirical estimate, this translates to $l^*(\theta) > \$50$ by equation (1). In other words, all individuals who take out loans of size \$50 or greater must have $G > L$. Setting $\alpha = 0.0005$, which is more than ten times smaller than our empirical estimates,

implies that $l^*(\theta) > \$200$.

G Details on Simulations

G.1 Association between liquid assets and payday loan bans

Table A5: **Do People Keep More Liquid Assets when Payday Lending is Banned?**

	(1) $ln(1+ assets)$	(2) $ln(1+ assets)$	(3) $1(assets \geq \$400)$	(4) $1(assets \geq \$400)$	(5) $1(assets \geq \$1000)$	(6) $1(assets \geq \$1000)$
Payday loan ban	-0.02	0.02	-0.00	-0.01	-0.00	0.00
	(0.22)	(0.12)	(0.03)	(0.02)	(0.03)	(0.02)
Observations	43,416	40,007	43,416	40,007	43,416	40,007
Dependent variable mean	4.22	4.16	0.51	0.50	0.43	0.42
Household fixed effects	No	Yes	No	Yes	No	Yes

Notes: This table presents regressions of liquid assets on a state payday loan ban indicator and household controls using Panel Survey of Income Dynamics data from 2003–2017. Liquid assets is total amount in checking or savings accounts, money market funds, certificates of deposit, government bonds, or treasury bills, for the respondent and members of his or her household. Household controls are ln(income), ln(education years), ln(household size), and ln(age). Standard errors are clustered by state.

G.2 Setup and summary

G.2.1 Setup

To quantify the welfare effects of payday lending regulations, we now calibrate a parametric version of our borrowing and repayment model. We assume that the benefit from borrowing is $u(l,\nu) = \nu\left(1 - e^{-\alpha_0 l}\right)$, where α_0 is a curvature parameter and $\nu \sim Lognormal(\mu_\nu, \sigma_\nu^2)$. Higher ν implies higher absolute and marginal utility from borrowing. We truncate ν at the 95th percentile of its distribution so that high ν draws do not drive the welfare estimates.

The utility cost of repaying x in period t is $k(x,\theta_t,\eta_t) = (\theta_t + \eta_t)\left(e^{\alpha_1 x} - 1\right)$, where α_1 is a curvature parameter and $\theta \sim Beta(a_\theta, b_\theta)$. We use the beta distribution for two reasons. First, the distribution needs to have bounded support; thick-tailed distributions such as the lognormal generate reborrowing rates that are too low. Second, the flexibility of the beta distribution allows us to match reborrowing rates with different amounts of variance in θ, and thus to consider scenarios where bias has small or large effects on repayment costs. Less flexible distributions would create a false sense of certainty about welfare results. We assume that $\eta \in \{0, \bar{\eta}\}$, with $\eta = 0$ in period $t = 1$ with probability q, and with the probability of transitioning to a different state given by $1 - q$.

The default cost is $\chi = \chi_0\left(e^{\alpha_1(l+p)} - 1\right)$. This parameterization makes it more costly to default on a larger loan. Constant default costs across loan sizes would generate much higher default rates on larger loans, which would run counter to the cross-sectional pattern in our data and the quasi-experimental results in Dobbie and Skiba (2013). Default costs might be higher for larger loans

because the "guilt" costs are higher, because lenders have more incentive to work to collect larger loans, and because the costs from losing access to credit may be larger for people who borrow more.

G.2.2 Summary

We assume a 15 percent borrowing fee, so $p(l)/l = 0.15$. We set $\delta = 0.998$, as this implies a five percent annual discount rate for two week periods, corresponding to bi-weekly pay cycles.

As discussed in Section 8.2.2, the welfare gains from borrowing are increasing in the slopes of u' and C', or equivalently the curvature of u and C. We thus choose α_0 and α_1 to be conservatively lower than our empirical estimate of $\alpha \approx 0.0064$. We set $\alpha_1 = 0.002$ and use the estimates of $(\beta, \tilde{\beta}) = (0.74, 0.77)$ from the second row of Table 3. We numerically verify that this produces $\tilde{C}$ with a coefficient of absolute risk aversion of almost exactly $\alpha = 0.002$, so these assumptions are internally consistent. We allow α_0 to range between the four non-zero curvature values considered in Table 3.

We calibrate the remaining parameters to match four moments from a random sample of borrowers who took out a loan from the Lender in 2017: the probability of reborrowing, the probability of defaulting, and the mean and variance of loan size. Panel (a) of Table A6 presents those four moments. Ideally, we would match the loan size distribution that would exist without a loan size cap, but only three states (Texas, Wyoming, and Utah) do not have loan size caps. To ensure a more representative sample of states while keeping the calibration simple, we use data from the 11 states where the Lender operates that have loan size caps between \$450 and \$550.

We calibrate these remaining parameters in two steps. In the first step, we calibrate $\bar{\eta}$, χ_0, q, a_θ, and b_θ. We set $\chi = 1.1$ to guarantee that borrowers never choose to default when $\eta = 0$ for any distribution of θ. We set $\bar{\eta}$ high enough such that borrowers always choose to default when $\eta = \bar{\eta}$ for any distribution of θ. This approach simplifies estimation by assuming that all borrowers default if and only if they draw a bad state $\bar{\eta}$. We then set $1 - q$ to match the empirical default rate of 0.028.

We then set the distribution of θ to match the empirical reborrowing probability. We set $\theta \sim Beta(a_\theta, 1)$, where a_θ is the only free parameter. This allows a family of distributions that spans everything between a uniform distribution ($a_\theta = 1$) and a degenerate distribution with no variance in θ ($a_\theta \to \infty$), which matches the limit case of vanishing volatility considered in Proposition 6.[37] In Appendix H we also consider a second scenario with $\theta \sim Beta(a_\theta, 0.02)$, which allows a highly bimodal distribution of θ, as in the limit case of high volatility considered in Proposition 5. In both scenarios, reborrowing probabilities are monotone in a_θ, and it is straightforward to find the a_θ that matches the empirical reborrowing probability.

With these parameters in hand, we numerically calculate perceived and actual expected loan repayment cost $\tilde{C}(l)$ and $C(l)$ for all l.

The second step of the calibration procedure is to calibrate the distribution of ν. To do so, we

[37] Changing the second scale parameter b_θ from 1 to values of 2, 3, 4, or 5 does not have a meaningful effect on the results.

simulate a set of potential borrowing spells, each with a draw of ν, and find the perceived optimal loan size $l^* \in [0, \$500]$ for each spell as a function of ν and $\tilde{C}(l)$. We cap loan sizes at \$500 to match the fact that our empirical data are drawn from states with loan size caps around \$500. We find the mean and variance (μ_ν, σ_ν^2) such that the distribution of simulated l^* (conditional on $l^* > 0$) matches the empirical mean and variance of loan sizes.

We simulate welfare under counterfactual policies for an exogenous set of potential borrowing spells. Because the distribution of ν is held fixed across counterfactuals, our simulations do not capture the possibility that rollover restrictions might result in more potential borrowing spells by breaking up single long spells into multiple short spells, or that people might keep larger buffer stocks in response to payday borrowing restrictions. This may be realistic: Appendix Table A5 shows that in the Panel Survey of Income Dynamics, households do not hold more liquid assets in states with payday loan bans or in years after their state imposes a ban.

Panel (b) of Table A6 presents the simulation parameters. Column 1 presents the calibration when $\alpha_0 = 0.002$, and column 2 presents the calibration when $\alpha_0 = 0.0002$.

The rest of this appendix provides more details on the calibration procedure.

Table A6: **Empirical Moments and Calibrated Parameters**

(a) **Empirical Moments**

Moment	Value
Probability of reborrowing	0.80
Probability of default	0.03
Mean loan amount	393
Standard deviation of loan amount	132

(b) **Simulation Parameters**

Parameter	(1) Higher demand elasticity	(2) Lower demand elasticity
α_0	0.0020	0.0002
α_1	0.002	0.002
δ	0.998	0.998
β	0.74	0.74
$\tilde{\beta}$	0.77	0.77
q	0.97	0.97
χ_0	1.10	1.10
$E[\theta]$	0.83	0.83
$Var[\theta]$	0.020	0.020
$E[\nu]$	1.99	3.46
$Var[\nu]$	0.92	0.31

Notes: Panel (a) presents the empirical moments that we match in our calibrated simulations. These moments are from all loans taken out in 2017 by a random sample of the Lender's customers in the 11 states where they operate that have loan size caps between \$450 and \$550. Panel (b) presents the simulation parameters we use. Column 1 is calibrated assuming $\alpha_0 = 0.002$. Column 2 is calibrated assuming $\alpha_0 = 0.0002$.

G.3 Solving the model for $T = \infty$

Let F be the CDF of θ_t. Let $r_t(\eta)$ be the actual expected utility of starting out in debt in period t given a realization of η. Let $\tilde{r}_t$ be the perceived utility cost, from the period $\tau < t$ perspective, of starting out in debt in period t. Define $\theta^\dagger(\eta)$ as the cutoff value such that a borrower repays in period t if and only if $\theta_t \leq \theta^\dagger$ (conditional on not defaulting). Define $\tilde{\theta}^\dagger$ to be the perceived cutoff in period $\tau < t$. Define $d(\eta)$ as the cutoff value such that a borrower defaults in period t if and only if $\theta_t > d(\eta)$. Define $\tilde{d}(\eta)$ to be the perceived cutoff in period $\tau < t$.

G.3.1 Perceived equilibrium

We look for a solution in which when things are good ($\eta = \underline{\eta}$), the borrower does not default (at baseline $\beta, \tilde{\beta}$ parameters) but when things are bad ($\eta = \bar{\eta}$) the person always defaults.

When $\eta = \underline{\eta}$ and the borrower is debating whether to repay in full or reborrow, she compares the repayment cost of paying in full, $(\theta_t + \underline{\eta})(e^{\alpha(l+p(l))} - 1)$, and the perceived repayment cost of reborrowing, $(\theta_t + \underline{\eta})(e^{\alpha p(l)} - 1) + \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))$. The θ_t where these two perceived repayment costs are equal defines $\tilde{\theta}^\dagger(\underline{\eta})$. When $\eta = \bar{\eta}$, $\tilde{\theta}^\dagger(\bar{\eta})$ is obtained similarly.

Thus, we have that the reborrowing cutoffs (conditional on not defaulting) are

$$\tilde{\theta}^\dagger(\underline{\eta}) = \frac{\tilde{\beta}\delta\left(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta})\right)}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} \tag{110}$$

$$\tilde{\theta}^\dagger(\bar{\eta}) = \frac{\tilde{\beta}\delta\left(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta})\right)}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} - \bar{\eta}. \tag{111}$$

Derivation of $\tilde{\theta}^\dagger(\underline{\eta})$:

$$(\theta + \underline{\eta})(e^{\alpha(l+p(l))} - 1) = (\theta + \underline{\eta})(e^{\alpha p(l)} - 1) + \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))$$

$$(\theta + \underline{\eta})(e^{\alpha(l+p(l))} - e^{\alpha p(l)}) = \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))$$

$$\theta + \underline{\eta} = \frac{\tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}$$

$$\theta = \frac{\tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}, \tag{112}$$

where the last line follows by the assumption that $\underline{\eta} = 0$.

Derivation of $\tilde{\theta}^\dagger(\bar{\eta})$:

$$(\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1) = (\theta + \bar{\eta})(e^{\alpha p(l)} - 1) + \tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))$$

$$(\theta+\bar{\eta})(e^{\alpha(l+p(l))}-e^{\alpha p(l)})=\tilde{\beta}\delta(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta}))$$

$$\theta+\bar{\eta}=\frac{\tilde{\beta}\delta(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha(l+p(l))}-e^{\alpha p(l)}}$$

$$\theta=\frac{\tilde{\beta}\delta(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha(l+p(l))}-e^{\alpha p(l)}}-\bar{\eta}. \tag{113}$$

When $\eta=\underline{\eta}$ there are two cases to consider. In the first case, χ is "large enough" so that $\tilde{\theta}^{\dagger}(\underline{\eta})<\tilde{d}(\underline{\eta})$ (i.e. χ is large enough to fit with the solution we are looking for). In this case, when the borrower debates between reborrowing and defaulting, she compares the perceived repayment cost of reborrowing, $(\theta_t+\underline{\eta})(e^{\alpha p(l)}-1)+\tilde{\beta}\delta(q\tilde{r}(\underline{\eta})+(1-q)\tilde{r}(\bar{\eta}))$, and the cost of defaulting, χ. The θ_t where these two perceived costs are equal defines $\tilde{d}(\underline{\eta})$ in this case. In the second case, χ is "small enough" so that $\tilde{\theta}^{\dagger}(\underline{\eta})>\tilde{d}(\underline{\eta})$ (which does not align with the solution we are looking for, but is needed for completeness). In this case, the borrower debates between repaying and defaulting. She compares the repayment cost of paying in full, $(\theta_t+\underline{\eta})(e^{\alpha(l+p(l))}-1)$, and the cost of defaulting, χ. The θ_t where these two costs are equal defines $\tilde{d}(\underline{\eta})$ in this case. Define $\tilde{\chi}^{\dagger}(\underline{\eta})$ as the boundary between these two cases. At $\tilde{\chi}^{\dagger}(\underline{\eta})$, we have that $\tilde{\theta}^{\dagger}(\underline{\eta})=\tilde{d}(\underline{\eta})$ (Note that when $\theta_t=\tilde{\theta}^{\dagger}(\underline{\eta})=\tilde{d}(\underline{\eta})$, the borrower is indifferent between repaying, reborrowing, and defaulting). Setting $\tilde{\theta}^{\dagger}(\underline{\eta})=\tilde{d}(\underline{\eta})$ and solving for χ defines $\tilde{\chi}^{\dagger}(\underline{\eta})$.

Similarly, when $\eta=\bar{\eta}$ there are two cases to consider. In the first case, χ is "small enough" so that $\tilde{\theta}^{\dagger}(\bar{\eta})>\tilde{d}(\bar{\eta})$ (i.e. χ is small enough to fit with the solution we are looking for). In this case, the borrower debates between repaying and defaulting. She compares the repayment cost of paying in full, $(\theta_t+\bar{\eta})(e^{\alpha(l+p(l))}-1)$, and the cost of defaulting, χ. The θ_t where these two costs are equal defines $\tilde{d}(\bar{\eta})$ in this case. In the second case, χ is "large enough" so that $\tilde{\theta}^{\dagger}(\bar{\eta})<\tilde{d}(\bar{\eta})$ (which does not align with the solution we are looking for, but is needed for completeness). In this case, when the borrower debates between reborrowing and defaulting, she compares the perceived repayment cost of reborrowing, $(\theta_t+\bar{\eta})(e^{\alpha p(l)}-1)+\tilde{\beta}\delta(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta}))$, and the cost of defaulting, χ. The θ_t where these two perceived costs are equal defines $\tilde{d}(\bar{\eta})$ in this case. Define $\tilde{\chi}^{\dagger}(\bar{\eta})$ as the boundary between these two cases. At $\tilde{\chi}^{\dagger}(\bar{\eta})$, we have that $\tilde{\theta}^{\dagger}(\bar{\eta})=\tilde{d}(\bar{\eta})$ (Note that when $\theta_t=\tilde{\theta}^{\dagger}(\bar{\eta})=\tilde{d}(\bar{\eta})$, the borrower is indifferent between repaying, reborrowing, and defaulting). Setting $\tilde{\theta}^{\dagger}(\bar{\eta})=\tilde{d}(\bar{\eta})$ and solving for χ defines $\tilde{\chi}^{\dagger}(\bar{\eta})$.

Thus, we have that the defaulting cutoffs are:

$$\tilde{d}(\underline{\eta})=\begin{cases}\frac{\chi}{e^{\alpha(l+p(l))}-1} & \text{if } \chi\leq\tilde{\chi}^{\dagger}(\underline{\eta})\\ \frac{\chi-\tilde{\beta}\delta\left(q\tilde{r}(\underline{\eta})+(1-q)\tilde{r}(\bar{\eta})\right)}{e^{\alpha p(l)}-1} & \text{if } \chi>\tilde{\chi}^{\dagger}(\underline{\eta})\end{cases} \tag{114}$$

$$\tilde{d}(\bar{\eta})=\begin{cases}\frac{\chi}{e^{\alpha(l+p(l))}-1}-\bar{\eta} & \text{if } \chi\leq\tilde{\chi}^{\dagger}(\bar{\eta})\\ \frac{\chi-\tilde{\beta}\delta\left(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta})\right)}{e^{\alpha p(l)}-1}-\bar{\eta} & \text{if } \chi>\tilde{\chi}^{\dagger}(\bar{\eta})\end{cases}, \tag{115}$$

where

$$\tilde{\chi}^{\dagger}(\underline{\eta}) = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} \tilde{\beta}\delta \left(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}) \right) \tag{116}$$

$$\tilde{\chi}^{\dagger}(\bar{\eta}) = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} \tilde{\beta}\delta \left(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}) \right). \tag{117}$$

Derivation of $\tilde{d}(\underline{\eta})$, $\chi \leq \tilde{\chi}^{\dagger}(\underline{\eta})$ case:

$$(\theta + \underline{\eta})(e^{\alpha(l+p(l))} - 1) = \chi$$

$$\theta + \underline{\eta} = \frac{\chi}{e^{\alpha(l+p(l))} - 1}$$

$$\theta = \frac{\chi}{e^{\alpha(l+p(l))} - 1} \tag{118}$$

Derivation of $\tilde{d}(\underline{\eta})$, $\chi > \tilde{\chi}^{\dagger}(\underline{\eta})$ case:

$$(\theta + \underline{\eta})(e^{\alpha p(l)} - 1) + \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta})) = \chi$$

$$(\theta + \underline{\eta})(e^{\alpha p(l)} - 1) = \chi - \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))$$

$$\theta + \underline{\eta} = \frac{\chi - \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))}{e^{\alpha p(l)} - 1}$$

$$\theta = \frac{\chi - \tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))}{e^{\alpha p(l)} - 1} \tag{119}$$

Derivation of $\tilde{d}(\bar{\eta})$, $\chi \leq \tilde{\chi}^{\dagger}(\bar{\eta})$ case:

$$(\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1) = \chi$$

$$\theta + \bar{\eta} = \frac{\chi}{e^{\alpha(l+p(l))} - 1}$$

$$\theta = \frac{\chi}{e^{\alpha(l+p(l))} - 1} - \bar{\eta} \tag{120}$$

Derivation of $\tilde{d}(\bar{\eta})$, $\chi > \tilde{\chi}^{\dagger}(\bar{\eta})$ case:

$$(\theta + \bar{\eta})(e^{\alpha p(l)} - 1) + \tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta})) = \chi$$

$$(\theta + \bar{\eta})(e^{\alpha p(l)} - 1) = \chi - \tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))$$

$$\theta + \bar{\eta} = \frac{\chi - \tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha p(l)} - 1}$$

$$\theta = \frac{\chi - \tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha p(l)} - 1} - \bar{\eta} \tag{121}$$

Derivation of $\tilde{\chi}^{\dagger}(\underline{\eta})$:

$$\tilde{\theta}^{\dagger}(\underline{\eta}) = \frac{\tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} = \frac{\chi}{e^{\alpha(l+p(l))} - 1} = \tilde{d}(\underline{\eta})$$

$$\chi = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}\tilde{\beta}\delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta})) \tag{122}$$

Derivation of $\tilde{\chi}^{\dagger}(\bar{\eta})$:

$$\tilde{\theta}^{\dagger}(\bar{\eta}) = \frac{\tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} - \bar{\eta} = \frac{\chi}{e^{\alpha(l+p(l))} - 1} - \bar{\eta} = \tilde{d}(\bar{\eta})$$

$$\frac{\tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} = \frac{\chi}{e^{\alpha(l+p(l))} - 1}$$

$$\chi = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}\tilde{\beta}\delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta})) \tag{123}$$

The Bellman operator on the continuation value functions is

$$B_1(\tilde{r}(\underline{\eta}), \tilde{r}(\bar{\eta})) \begin{cases} \int_{\underbrace{\theta \leq \tilde{d}(\underline{\eta})}_{Repay}} \theta(e^{\alpha(l+p(l))} - 1)dF + \chi \overbrace{(1 - F(\tilde{d}(\underline{\eta})))}^{Pr(Default)} & \text{if } \chi \leq \tilde{\chi}^{\dagger}(\underline{\eta}) \\ \overbrace{(1 - F(\tilde{d}(\underline{\eta})))}^{Pr(Default)} \chi + \int_{\underbrace{\theta \leq \tilde{\theta}^{\dagger}(\underline{\eta})}_{Repay}} \theta(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\underbrace{\tilde{\theta}^{\dagger}(\underline{\eta}) \leq \theta \leq \tilde{d}(\underline{\eta})}_{Reborrow}} [\theta(e^{\alpha p(l)} - 1) + \delta(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta}))]dF & \text{if } \chi > \tilde{\chi}^{\dagger}(\underline{\eta}) \end{cases} \tag{124}$$

$$B_2(\tilde{r}(\underline{\eta}), \tilde{r}(\bar{\eta})) = \begin{cases} \int_{\underbrace{\theta \leq \tilde{d}(\bar{\eta})}_{Repay}} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \chi \overbrace{(1 - F(\tilde{d}(\bar{\eta})))}^{Pr(Default)} & \text{if } \chi \leq \tilde{\chi}^{\dagger}(\bar{\eta}) \\ \overbrace{(1 - F(\tilde{d}(\bar{\eta})))}^{Pr(Default)} \chi + \int_{\underbrace{\theta \leq \tilde{\theta}^{\dagger}(\bar{\eta})}_{Repay}} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\underbrace{\tilde{\theta}^{\dagger}(\bar{\eta}) \leq \theta \leq \tilde{d}(\bar{\eta})}_{Reborrow}} [(\theta + \bar{\eta})(e^{\alpha p(l)} - 1) + \delta(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta}))]dF & \text{if } \chi > \tilde{\chi}^{\dagger}(\bar{\eta}) \end{cases} \tag{125}$$

The solution is a fixed point of $B = (B_1, B_2)$: $B_1(\tilde{r}) = \tilde{r}(\underline{\eta})$, $B_2(\tilde{r}) = \tilde{r}(\bar{\eta})$. For a given set of parameters χ, q, a, and b and $\bar{\eta}$, we can solve Equations (124) and (125) by plugging in Equations (110), (111), (114), and (115) into Equations (124) and (125), which gives a set of two equations in two unknowns for each of the three cases (i) $\tilde{d}(\underline{\eta}) < \tilde{\theta}^{\dagger}(\underline{\eta})$, $\tilde{d}(\bar{\eta}) < \tilde{\theta}^{\dagger}(\bar{\eta})$, or (ii) $\tilde{d}(\underline{\eta}) > \tilde{\theta}^{\dagger}(\underline{\eta})$, $\tilde{d}(\bar{\eta}) < \tilde{\theta}^{\dagger}(\bar{\eta})$, or (iii) $\tilde{d}(\underline{\eta}) > \tilde{\theta}^{\dagger}(\underline{\eta})$, $\tilde{d}(\bar{\eta}) > \tilde{\theta}^{\dagger}(\bar{\eta})$. We solve for the parameters in each case, and then check whether the solution satisfies the condition of that case. As shown in Theorem 1, the solution is unique.

Once $\tilde{r}(\underline{\eta})$ and $\tilde{r}(\bar{\eta})$ are computed, we can immediately back out $\tilde{\theta}^{\dagger}(\underline{\eta}), \tilde{\theta}^{\dagger}(\bar{\eta}), \tilde{d}(\underline{\eta}), \tilde{d}(\bar{\eta})$ from Equations (110), (111), (114), and (115).

G.3.2 Actual loan repayment behavior

Now actual behavior can be obtained by replacing $\tilde{\beta}$ with β in the preceding equations and is given as follows:

1. When $\eta_t = \underline{\eta}$: (a) if $\chi > \chi^{\dagger}(\underline{\eta})$, the person defaults if $\theta_t > d(\underline{\eta})$, repays that period if $\theta_t \leq \theta^{\dagger}(\underline{\eta})$, and otherwise just continues on to period t after only paying the fee $p(l)$. (b) if $\chi \leq \chi^{\dagger}(\underline{\eta})$, the person defaults if $\theta_t > d(\underline{\eta})$ and repays that period if $\theta_t \leq d(\underline{\eta})$.

2. When $\eta_t = \bar{\eta}$: (a) if $\chi \leq \chi^\dagger(\bar{\eta})$, the person defaults if $\theta_t > d(\bar{\eta})$ and repays that period if $\theta_t \leq d(\bar{\eta})$. (a) if $\chi > \chi^\dagger(\bar{\eta})$, the person defaults if $\theta_t > d(\bar{\eta})$, repays that period if $\theta_t \leq \theta^\dagger(\bar{\eta})$, and otherwise just continues on to period t after only paying the fee $p(l)$.

Where (all derivations are the same as before, but with β replacing $\tilde{\beta}$):

$$\theta^\dagger(\underline{\eta}) = \frac{\beta\delta\left(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta})\right)}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} \tag{126}$$

$$\theta^\dagger(\bar{\eta}) = \frac{\beta\delta\left(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta})\right)}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}} - \bar{\eta} \tag{127}$$

$$d(\underline{\eta}) = \begin{cases} \frac{\chi}{e^{\alpha(l+p(l))}-1} & \text{if } \chi \leq \chi^\dagger(\underline{\eta}) \\ \frac{\chi - \beta\delta\left(q\tilde{r}(\underline{\eta})+(1-q)\tilde{r}(\bar{\eta})\right)}{e^{\alpha p(l)}-1} & \text{if } \chi > \chi^\dagger(\underline{\eta}) \end{cases} \tag{128}$$

$$d(\bar{\eta}) = \begin{cases} \frac{\chi}{e^{\alpha(l+p(l))}-1} - \bar{\eta} & \text{if } \chi \leq \chi^\dagger(\bar{\eta}) \\ \frac{\chi - \beta\delta\left(q\tilde{r}(\bar{\eta})+(1-q)\tilde{r}(\underline{\eta})\right)}{e^{\alpha p(l)}-1} - \bar{\eta} & \text{if } \chi > \chi^\dagger(\bar{\eta}) \end{cases} \tag{129}$$

$$\chi^\dagger(\underline{\eta}) = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}\beta\delta\left(q\tilde{r}(\underline{\eta}) + (1-q)\tilde{r}(\bar{\eta})\right) \tag{130}$$

$$\chi^\dagger(\bar{\eta}) = \frac{e^{\alpha(l+p(l))} - 1}{e^{\alpha(l+p(l))} - e^{\alpha p(l)}}\beta\delta\left(q\tilde{r}(\bar{\eta}) + (1-q)\tilde{r}(\underline{\eta})\right) \tag{131}$$

G.3.3 Objective function for taking out a loan

In period 0, the person's perceived cost of taking out the loan is $\tilde{r}(l, \eta)$, and thus the person chooses l to maximize one of the following two objective functions

$$\max_{l\in[0,\bar{l}]} \beta\left[1 - \nu e^{-\alpha l} - \tilde{C}(l)\right] \tag{132}$$

$$\max_{l\in[0,\bar{l}]} 1 - \nu e^{-\alpha l} - \beta\tilde{C}(l), \tag{133}$$

where $\tilde{C}(l) = q\tilde{r}(l, \underline{\eta}) + (1-q)\tilde{r}(l, \bar{\eta})$. The first objective function corresponds to the benefits of the loan being realized in the future (e.g., car repair), while the second objective function corresponds to the loan being used for immediate consumption.

G.3.4 Borrower welfare

We adopt the time $t = 0$ criterion to compute borrower welfare. The decision rule in sub-section G.3.2 leads to the following equations for the continuation value function, where $d(\eta)$ and $\theta^\dagger(\eta)$ are as defined in that sub-section:

$$r(\underline{\eta}) = \begin{cases} \int_{\theta \leq d(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\underline{\eta}))) & \text{if } \chi \leq \chi^\dagger(\underline{\eta}) \\ \quad (1 - F(d(\underline{\eta})))\chi + \int_{\theta \leq \theta^\dagger(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\theta^\dagger(\underline{\eta}) \leq \theta \leq d(\underline{\eta})} [\theta(e^{\alpha p(l)} - 1) + \delta(qr(\underline{\eta}) + (1-q)r(\bar{\eta}))]dF & \text{if } \chi > \chi^\dagger(\underline{\eta}) \end{cases} \tag{134}$$

$$r(\bar{\eta}) = \begin{cases} \int_{\theta \leq d(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\bar{\eta}))) & \text{if } \chi \leq \chi^\dagger(\bar{\eta}) \\ \quad (1 - F(d(\bar{\eta})))\chi + \int_{\theta \leq \theta^\dagger(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\theta^\dagger(\bar{\eta}) \leq \theta \leq d(\bar{\eta})} [(\theta + \bar{\eta})(e^{\alpha p(l)} - 1) + \delta(qr(\bar{\eta}) + (1-q)r(\underline{\eta}))]dF & \text{if } \chi > \chi^\dagger(\bar{\eta}) \end{cases} \tag{135}$$

So once we know $d(\eta)$ and $\theta^\dagger(\eta)$, we just have two linear equations in two unknowns that we can immediately use to solve for $r(\underline{\eta})$ and $r(\bar{\eta})$. Note that $\chi^\dagger(\underline{\eta}) \leq \chi^\dagger(\bar{\eta})$, because $\tilde{r}(\underline{\eta}) \leq \tilde{r}(\bar{\eta})$. To see how to solve this linear system of equations, define constants

$$A \equiv \int_{\theta \leq d(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\underline{\eta}))), \tag{136}$$

$$B \equiv (1 - F(d(\underline{\eta})))\chi + \int_{\theta \leq \theta^\dagger(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \int_{\theta^\dagger(\underline{\eta}) \leq \theta \leq d(\underline{\eta})} \theta(e^{\alpha p(l)} - 1)dF, \tag{137}$$

$$C \equiv \delta q[F(d(\underline{\eta})) - F(\theta^\dagger(\underline{\eta}))], \tag{138}$$

$$D \equiv \delta(1-q)[F(d(\underline{\eta})) - F(\theta^\dagger(\underline{\eta}))]. \tag{139}$$

Then, if $\chi \leq \chi^\dagger(\underline{\eta})$, note that:

$$r(\underline{\eta}) = \int_{\theta \leq d(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\underline{\eta}))) = A. \tag{140}$$

If $\chi > \chi^\dagger(\underline{\eta})$, note that:

$$\begin{aligned} r(\underline{\eta}) &= (1 - F(d(\underline{\eta})))\chi + \int_{\theta \leq \theta^\dagger(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \int_{\theta^\dagger(\underline{\eta}) \leq \theta \leq d(\underline{\eta})} \theta(e^{\alpha p(l)} - 1)dF \\ &+ \delta q[F(d(\underline{\eta})) - F(\theta^\dagger(\underline{\eta}))]r(\underline{\eta}) + \delta(1-q)[F(d(\underline{\eta})) - F(\theta^\dagger(\underline{\eta}))]r(\bar{\eta}) \\ &= B + Cr(\underline{\eta}) + Dr(\bar{\eta}). \end{aligned} \tag{141}$$

Define constants

$$G \equiv \int_{\theta \leq d(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\bar{\eta}))) \tag{142}$$

$$H \equiv (1 - F(d(\bar{\eta})))\chi + \int_{\theta \le \theta^\dagger(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \int_{\theta^\dagger(\bar{\eta}) \le \theta \le d(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha p(l)} - 1)dF \quad (143)$$

$$I \equiv \delta q[F(d(\bar{\eta})) - F(\theta^\dagger(\bar{\eta}))] \quad (144)$$

$$J \equiv \delta(1 - q)[F(d(\bar{\eta})) - F(\theta^\dagger(\bar{\eta}))]. \quad (145)$$

Then, if $\chi \le \chi^\dagger(\bar{\eta})$, note that:

$$r(\bar{\eta}) = \int_{\theta \le d(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d(\bar{\eta}))) = G. \quad (146)$$

If $\chi > \chi^\dagger(\bar{\eta})$, note that:

$$\begin{aligned} r(\bar{\eta}) &= (1 - F(d(\bar{\eta})))\chi + \int_{\theta \le \theta^\dagger(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \int_{\theta^\dagger(\bar{\eta}) \le \theta \le d(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha p(l)} - 1)dF \\ &\quad + \delta q[F(d(\bar{\eta})) - F(\theta^\dagger(\bar{\eta}))]r(\bar{\eta}) + \delta(1 - q)[F(d(\bar{\eta})) - F(\theta^\dagger(\bar{\eta}))]r(\underline{\eta}) \\ &= H + Ir(\bar{\eta}) + Jr(\underline{\eta}). \end{aligned} \quad (147)$$

So, if $\chi \le \chi^\dagger(\underline{\eta}) \le \chi^\dagger(\bar{\eta})$, we get the system:

$$r(\underline{\eta}) = A$$

$$r(\bar{\eta}) = G$$

If $\chi^\dagger(\underline{\eta}) < \chi \le \chi^\dagger(\bar{\eta})$, we get the system:

$$(1 - C)r(\underline{\eta}) - Dr(\bar{\eta}) = B$$

$$r(\bar{\eta}) = G$$

And, if $\chi^\dagger(\underline{\eta}) \le \chi^\dagger(\bar{\eta}) < \chi$, we get the system:

$$(1 - C)r(\underline{\eta}) - Dr(\bar{\eta}) = B$$

$$-Jr(\underline{\eta}) + (1 - I)r(\bar{\eta}) = H$$

Each system is easily solved.

Once we have $r(\underline{\eta})$ and $r(\bar{\eta})$, actual borrower welfare is going to be

$$E_\nu \left[1 - \nu e^{-\alpha l^*(\nu)} - C(l)\right] \tag{148}$$

where $C(l) = qr(l, \underline{\eta}) + (1-q)r(l, \bar{\eta})$ and $l^*(\nu)$ is the loan size given a realization of ν, and where we just set $r \equiv 0$ when $l^*(\nu) = 0$.

G.4 $T < \infty$

We use backwards induction to solve the finite-horizon model.

In period T, the agent must either repay or default. The cost of repaying is $(\theta_T + \eta)(e^{\alpha(l+p(l))} - 1)$ and the cost of defaulting is χ. Thus, the agent repays if

$$(\theta_T + \eta)(e^{\alpha(l+p(l))} - 1) \leq \chi$$

$$\theta_T \leq \frac{\chi}{e^{\alpha(l+p(l))} - 1} - \eta. \tag{149}$$

This gives us the cutoffs

$$d_T(\underline{\eta}) = \tilde{d}_T(\underline{\eta}) = \frac{\chi}{e^{\alpha(l+p(l))} - 1} \tag{150}$$

$$d_T(\bar{\eta}) = \tilde{d}_T(\bar{\eta}) = \frac{\chi}{e^{\alpha(l+p(l))} - 1} - \bar{\eta}. \tag{151}$$

Which gives us the expected costs

$$r_T(\underline{\eta}) = \tilde{r}_T(\underline{\eta}) = (1 - F(d_T(\underline{\eta})))\chi + \int_{\theta \leq d_T(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF \tag{152}$$

$$r_T(\bar{\eta}) = \tilde{r}_T(\bar{\eta}) = (1 - F(d_T(\bar{\eta})))\chi + \int_{\theta \leq d_T(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF. \tag{153}$$

Note that we can calculate these directly once we have the calibrated parameters of the model (which we'll get from calibrating the infinite-time model).

In period $T-1$, the agent has the option to repay, reborrow, or default, as in the infinite-horizon model. The recursive formulas in section G.3 for the perceived and actual cutoffs and expected costs hold here, as well. We plug in $r_T(\underline{\eta})$ and $r_T(\bar{\eta})$ into the right-hand side of Equations (110),(111),(114),(115) to derive the perceived cutoffs in $T-1$, which we then use to calculate $\tilde{r}_{T-1}$ using Equations (124) and (125) and $\tilde{r}_T$. From this we obtain the actual period $T-1$ cutoffs using Equations (126),(127),(128), and (129), which then give us r_{T-1} through the recursion

$$r_{t-1}(\underline{\eta}) = \begin{cases} \int_{\theta \le d_t(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d_t(\underline{\eta}))) & \text{if } \chi \le \chi^{\dagger}(\underline{\eta}) \\ \quad (1 - F(d(_t\underline{\eta})))\chi + \int_{\theta \le \theta_t^{\dagger}(\underline{\eta})} \theta(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\theta_t^{\dagger}(\underline{\eta}) \le \theta \le d_t(\underline{\eta})} [\theta(e^{\alpha p(l)} - 1) + \delta(q r_t(\underline{\eta}) + (1-q) r_t(\bar{\eta}))]dF & \text{if } \chi > \chi^{\dagger}(\underline{\eta}) \end{cases} \tag{154}$$

$$r_{t-1}(\bar{\eta}) = \begin{cases} \int_{\theta \le d_t(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF + \chi(1 - F(d_t(\bar{\eta}))) & \text{if } \chi \le \chi^{\dagger}(\bar{\eta}) \\ \quad (1 - F(d_t(\bar{\eta})))\chi + \int_{\theta \le \theta_t^{\dagger}(\bar{\eta})} (\theta + \bar{\eta})(e^{\alpha(l+p(l))} - 1)dF & \\ + \int_{\theta_t^{\dagger}(\bar{\eta}) \le \theta \le d_t(\bar{\eta})} [(\theta + \bar{\eta})(e^{\alpha p(l)} - 1) + \delta(q r_t(\bar{\eta}) + (1-q) r_t(\underline{\eta}))]dF & \text{if } \chi > \chi^{\dagger}(\bar{\eta}) \end{cases} \tag{155}$$

We then use $\tilde{r}_{T-1}(\underline{\eta})$, $r_{T-1}(\underline{\eta})$, $\tilde{r}_{T-1}(\bar{\eta})$, and $r_{T-1}(\bar{\eta})$ to calculate $\tilde{r}_{T-2}(\underline{\eta})$, $r_{T-2}(\underline{\eta})$, $\tilde{r}_{T-2}(\bar{\eta})$, and $r_{T-2}(\bar{\eta})$. We continue this recursion until we have calculated $\tilde{r}_1(\underline{\eta})$, $r_1(\underline{\eta})$, $\tilde{r}_1(\bar{\eta})$, and $r_1(\bar{\eta})$.

If $\tilde{C}(l) = q\tilde{r}_1(l, \underline{\eta}) + (1-q)\tilde{r}_1(l, \bar{\eta})$ and $C(l) = q r_1(l, \underline{\eta}) + (1-q) r_1(l, \bar{\eta})$, then the agent solves either Equation (132) or Equation (133) and we can calculate welfare as $E_\nu \left[1 - \nu e^{-\alpha l^*(\nu)} - C(l)\right]$, where $l^*(\nu)$ is the loan size given a realization of ν, and where we just set $r \equiv 0$ when $l^*(\nu) = 0$.

G.5 Learning

In period 0, the agent thinks they'll act with $\tilde{\beta} = \tilde{\beta}_0$ in all future periods. Thus, we can calculate $\tilde{C}(l)$ as in section G.3.1, but with $\tilde{\beta} = \tilde{\beta}_0$. In period $t = 4$, the agent has $\tilde{\beta} = \beta$. By setting $\tilde{\beta} = \beta$, we can calculate $r(\underline{\eta})$ and $r(\bar{\eta})$ as in section G.3. Call these $r_4(\underline{\eta})$ and $r_4(\bar{\eta})$.

Now consider period $t = 3$. In this period, the agent thinks that they'll have $\tilde{\beta} = \tilde{\beta}_0$ in period 4 and onwards. Thus, we can use our fixed-point solutions for $\tilde{r}(\underline{\eta})$ and $\tilde{r}(\bar{\eta})$ to calculate the actual cutoffs (using the formulas in section G.3.2). Then, we can use these actual cutoffs, $r_4(\underline{\eta})$, and $r_4(\bar{\eta})$ to calculate $r_3(\underline{\eta})$ and $r_3(\bar{\eta})$ (using the formulas in section G.3.2).

We can continue in this way to calculate $r_2(\underline{\eta})$ and $r_2(\bar{\eta})$ and then again to calculate $r_1(\underline{\eta})$ and $r_1(\bar{\eta})$. With those in hand, we have $C(l)$ for welfare purposes.

G.6 Details on numerical procedures

As described in Section 8.3, we calibrate our parametric model of borrowing and repayment in two steps. In the first step, we calibrate the scale parameters of the beta distribution and the transition probability $1 - q$ to match the empirical rate of reborrowing (0.8) and the empirical default rate (0.028) respectively. In the second step, we calibrate the distribution of $\nu \sim lognormal(\mu_\nu, \sigma_\nu^2)$ to match the empirical mean (393) and standard deviation (132) of loan sizes.

Our first step calibration procedure is as follows: first, given a choice of scale parameters of the beta distribution, the coefficient of absolute risk aversion α_1, the transition probability, and the free variables χ and $\bar{\eta}$, we can solve the continuation values $\tilde{r}(\underline{\eta})$ and $\tilde{r}(\bar{\eta})$ for any loan amount l by substituting Equations (110) - (117) into Equations (124) and (125), producing a

system of two equations with two unknowns. We then solve this system via fixed point iteration. Given a simulated borrower's loan amount and the corresponding perceived continuation values, we next simulate actual reborrowing behavior as described in Section G.3.2: in the initial period, borrowers have probability $1-q$ of being in state $\bar{\eta}$. After receiving a θ_t draw, borrowers can either repay, reborrow, or default, with the associated cutoff values coming from plugging the perceived continuation values into Equations (126) - (129). If borrowers do not repay or default, they switch states with probability $1-q$ and receive a new θ_t draw. This process repeats until the borrower repays or defaults, thus simulating an entire borrowing history given an initial choice of l.

To draw the loan amounts we use for the first-stage calibration, we sample 10,000 empirical loans from data provided by the Lender. We restrict our sample to the 11 states which have a loan cap between \$450 and \$550 and to loans that were originated in 2017, resulting in approximately $104,000$ loans that we sample from.[38] For each of the 10,000 loan amounts, we then simulate the entire borrowing history to estimate the simulated reborrowing and default rate.

To calibrate the parameters of the beta distribution, we first fix a given choice of the second scale parameter of the beta distribution and then successively refine a grid search over the first scale parameter of the beta distribution until our simulated reborrowing and default rates match their empirical counterparts. The grid search procedure is as follows: we start by searching over the first scale parameter in a grid of steps of size 0.5, ranging from 0 to 30. We then pick the interval that is closest to the empirical reborrowing probability and search in that interval in steps of size 0.1. As discussed in Section 8.3, when simulating reborrowing decisions, we assume that borrowers have an $\alpha_1 = 0.002$. We calibrate θ making three different assumptions: (1) for our primary estimates, we assume that $(\tilde{\beta}, \beta) = (0.77, 0.74)$ (corresponding to the $(\tilde{\beta}, \beta)$ we estimate when assuming $\alpha = 0.002$) and $\theta \sim Beta(a_\theta, 1)$, (2) for our bimodal estimates, we assume that $(\tilde{\beta}, \beta) = (0.77, 0.74)$ and $\theta \sim Beta(a_\theta, 0.02)$, and (3) for our expert forecasts, we assume that $(\tilde{\beta}, \beta) = (0.86, 0.63)$ (corresponding to the $(\tilde{\beta}, \beta)$ implied by our expert survey) and $\theta \sim Beta(a_\theta, 1)$.

Our second step calibration procedure is as follows: we calibrate the distribution of $\nu \sim Lognormal(\mu_\nu, \sigma_\nu^2)$ to match the empirical mean and standard deviation of loan sizes. Given a draw ν and a value of α_0, we find each simulated borrower's optimal loan size l^*, with a maximum loan size of \$500 to match the fact that our empirical data is drawn from states with loan size caps around \$500. We make four different assumptions about the value of α_0: $0.0002, 0.0005, 0.002$, or 0.0064. We then solve for (μ_ν, σ_ν^2) using the Nelder-Mead algorithm.

To estimate welfare under different policy counterfactuals and different values of α_0, we first draw 50,000 values of ν using the calibrated parameters above. For our baseline infinite horizon, no learning model, we back out the perceived continuation values again by substituting Equations (110) - (117) into Equations (124) and (125) and solving via fixed point iteration. To solve for the actual costs $C(l)$, we follow Section G.3.4. Each of the 50,000 simulated borrowers then choose

[38] There are 16 states which have a loan cap between \$450 and \$550: Alabama, Alaska, Colorado, Florida, Hawaii, Indiana, Iowa, Kansas, Kentucky, Mississippi, Missouri, North Dakota, Oklahoma, Rhode Island, South Carolina, and Virginia. However, our Lender does not have lending data in Alaska, Colorado, Hawaii, North Dakota, or Rhode Island, leading to our sample of 11 states.

an $l^* \in [0, \bar{l}]$ that solves Equation (132). With each borrower's choice of of l^* and associated cost $C(l^*)$, we estimate average borrower welfare using Equation (148).

When estimating welfare under a rollover restriction, we instead solve for $\tilde{C}(l)$ and $C(l)$ by backwards induction as described in Section G.4. This allows us to solve each borrower's choice of l^* using Equation (132) and calculate welfare as in Equation (148).

When estimating welfare assuming learning, we assume that borrowers have a constant $\beta = 0.74$. In periods 1 - 3, borrowers are partially naive and have a $\tilde{\beta} = \tilde{\beta}_0$. We compute $\tilde{\beta}_0$ by using the estimate of naivete among the subset of new borrowers who participated in our field experiment, where a new borrower is defined as a borrower who took out less than 4 payday loans from the Lender in the 6 months prior to the start date of our experiment. $\tilde{\beta}_0$ is then calculated as β divided by the estimated naivete of new borrowers, which in our primary estimates results in $\tilde{\beta}_0 = 0.88$. After period $t = 4$, borrowers become perfectly sophisticated. Using these parameters, we then compute the perceived and actual costs of borrowing. The process of computing $\tilde{C}(l)$ is the same as in the infinite-horizon case. To calculate $C(l)$, we follow the procedure described in Section G.5. In the fourth-period, since borrowers are sophisticated, the actual continuation values are equal to the perceived continuation values. Substituting the perceived continuation values into Equations (126) - (129) yields the actual cutoffs, which we can plug into the recursion in Equations (154)-(155) to yield the third period actual continuation values. Recursively repeating this process yields $C(l)$. Once we compute $\tilde{C}(l)$ and $C(l)$ for every l, we again solve each borrower's choice of l^* using Equation (132) and calculate welfare as in Equation (148).

When estimating welfare assuming heterogeneous borrowers using our primary estimates of $(\tilde{\beta}, \beta)$, we assume that 50% of borrowers are perfectly time consistent and that 50% of borrowers have $(\tilde{\beta}, \beta) = (0.53, 0.48)$ such that the average $(\tilde{\beta}, \beta)$ equals $(0.77, 0.74)$. Aggregate welfare in the heterogeneous case is thus the average of welfare for time-consistent borrowers and partially-naive borrowers. When estimating welfare assuming heterogeneous borrowers using experts' forecasts of $(\tilde{\beta}, \beta)$, we again assume that 50% of borrowers are perfectly time consistent and that 50% of borrowers have $(\tilde{\beta}, \beta) = (0.73, 0.26)$ such that the average $(\tilde{\beta}, \beta)$ equals experts' forecasts $(0.86, 0.63)$.

H Additional Simulation Results

Table A7 presents the results of our calibrations at our empirical estimates of β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 1)$. Table A8 presents the results of our calibrations at our empirical estimates of β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 0.02)$. Table A9 presents the results of our calibrations using experts' forecasts of β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 1)$. For our expert opinion calibration, we use the $\tilde{\beta} = 0.86$ forecasted by the average expert. We also estimate $\beta/\tilde{\beta} = 0.73$ by inserting experts' average forecast of borrower misprediction into Equation (42). We then back out the implied expert opinion of $\beta = 0.63$.

Tables A10 and A11 present the welfare estimates for $\alpha_0 = 0.0064$ and $\alpha_0 = 0.0005$, respectively, using our empirical estimates of β and $\tilde{\beta}$ and assuming $\theta \sim Beta(a_\theta, 1)$. Tables A12 - A15

present the welfare estimates for different values of α_0, using our empirical estimates of β and $\tilde{\beta}$ and assuming $\theta \sim Beta(a_\theta, 0.02)$. Lastly, Tables A16 - A19 present the welfare estimates for different values of α_0, using experts' forecasts of β and $\tilde{\beta}$ and assuming $\theta \sim Beta(a_\theta, 1)$.

Examining how the welfare costs of present focus change when we calibrate the model using experts' forecasts of β and $\tilde{\beta}$, column 1 of row 4 in Panel (b) of Table A17 shows that the welfare costs of present focus at experts' forecasted parameters are only 6 percent of time-consistent borrowers' surplus. Furthermore, the basic pattern of policy impacts is very similar to Panel (a) of Table 5: bans and loan size caps significantly reduce welfare, and in this case even a rollover restriction has a slightly negative effect.

Table A7: **Calibrated Parameters: Empirical Estimates,** $\theta \sim Beta(a_\theta, 1)$

Parameter	(1) $\alpha_0 = 0.0064$	(2) $\alpha_0 = 0.002$	(3) $\alpha_0 = 0.0005$	(4) $\alpha_0 = 0.0002$
$E[\theta]$	0.83	0.83	0.83	0.83
$Var[\theta]$	0.020	0.020	0.020	0.020
$E[\nu]$	2.86	1.99	2.69	3.46
$Var[\nu]$	3.91	0.92	0.41	0.31

Notes: This table presents the calibrated parameters for additional simulations using our empirical estimates of β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 1)$.

Table A8: **Calibrated Parameters: Empirical Estimates,** $\theta \sim Beta(a_\theta, 0.02)$

Parameter	(1) $\alpha_0 = 0.0064$	(2) $\alpha_0 = 0.002$	(3) $\alpha_0 = 0.0005$	(4) $\alpha_0 = 0.0002$
$E[\theta]$	0.82	0.82	0.82	0.82
$Var[\theta]$	0.134	0.134	0.134	0.134
$E[\nu]$	2.11	1.21	1.90	2.50
$Var[\nu]$	3.23	0.53	0.16	0.86

Notes: This table presents the calibrated parameters for additional simulations using our empirical estimates of β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 0.02)$.

Table A9: **Calibrated Parameters: Expert Forecasts,** $\theta \sim Beta(a_\theta, 1)$

Parameter	(1) $\alpha_0 = 0.0064$	(2) $\alpha_0 = 0.002$	(3) $\alpha_0 = 0.0005$	(4) $\alpha_0 = 0.0002$
$E[\theta]$	0.64	0.64	0.64	0.64
$Var[\theta]$	0.060	0.060	0.060	0.060
$E[\nu]$	2.30	1.43	2.16	2.62
$Var[\nu]$	3.63	0.85	1.08	2.39

Notes: This table presents the calibrated parameters for additional simulations using experts' beliefs about β and $\tilde{\beta}$, assuming $\theta \sim Beta(a_\theta, 1)$.

Table A10: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0064$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	398	0.42	481
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	393	0.78	610
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	410	0.79	640
4	Heterogeneous	390	0.66	678
5	$\beta = 0.74, \tilde{\beta} = 1$	398	0.85	714
6	$\beta = 0.74, \tilde{\beta} = 0.74$	392	0.76	593
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	397	0.83	679
8	Primary, heterogeneous, learning, consume in $t = 0$	409	0.68	899
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	396	0.91	899
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	420	0.91	955

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline (\$500 cap)	(2) \$400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	96.9%	100.0%	99.5%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	99.6%	96.6%	99.8%	99.1%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	99.6%	96.5%	99.7%	99.0%
4	Heterogeneous	99.1%	96.1%	99.7%	98.4%
5	$\beta = 0.74, \tilde{\beta} = 1$	99.2%	96.2%	99.7%	98.5%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	99.7%	96.6%	99.8%	99.1%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	99.5%	96.5%	99.7%	98.9%
8	Primary, heterogeneous, learning, consume in $t = 0$	99.1%	96.2%	99.7%	98.4%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	98.2%	95.4%	99.6%	97.2%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	98.0%	95.3%	99.5%	97.0%

Notes: Panel (a) presents simulated borrowing behavior under a \$500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a \$500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0064$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \leq t \leq 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \geq 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A11: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0005$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	409	0.43	495
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	394	0.78	611
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	440	0.79	692
4	Heterogeneous	382	0.67	653
5	$\beta = 0.74, \tilde{\beta} = 1$	409	0.85	735
6	$\beta = 0.74, \tilde{\beta} = 0.74$	391	0.76	591
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	404	0.83	693
8	Primary, heterogeneous, learning, consume in $t = 0$	435	0.69	979
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	403	0.91	917
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	464	0.91	1064

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	89.6%	99.6%	88.7%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	91.0%	82.4%	94.2%	79.2%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	88.9%	81.2%	92.5%	76.2%
4	Heterogeneous	80.5%	73.5%	94.0%	67.9%
5	$\beta = 0.74, \tilde{\beta} = 1$	80.0%	73.7%	93.5%	66.5%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	92.6%	83.7%	94.3%	80.9%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	89.0%	80.7%	93.8%	75.7%
8	Primary, heterogeneous, learning, consume in $t = 0$	76.9%	72.6%	90.5%	60.3%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	57.3%	55.1%	90.6%	39.6%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	46.2%	49.2%	86.5%	21.8%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0005$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \leq t \leq 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \geq 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A12: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0064$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	393	0.79	612
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	393	0.80	621
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	411	0.80	658
4	Heterogeneous	393	0.80	627
5	$\beta = 0.74, \tilde{\beta} = 1$	393	0.79	617
6	$\beta = 0.74, \tilde{\beta} = 0.74$	393	0.80	623
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	393	0.80	626
8	Primary, heterogeneous, learning, consume in $t = 0$	392	0.80	630
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	393	0.81	632
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	421	0.80	672

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	97.0%	98.4%	97.9%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	100.0%	97.0%	98.3%	97.9%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	99.9%	97.0%	98.3%	97.8%
4	Heterogeneous	99.9%	97.0%	98.3%	97.8%
5	$\beta = 0.74, \tilde{\beta} = 1$	100.0%	97.0%	98.3%	97.9%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	100.0%	97.0%	98.3%	97.9%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	100.0%	97.0%	98.3%	97.9%
8	Primary, heterogeneous, learning, consume in $t = 0$	99.7%	96.8%	98.1%	97.6%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	99.9%	97.0%	98.3%	97.8%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	99.9%	96.9%	98.2%	97.7%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0064$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \le t \le 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \ge 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A13: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.002$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	393	0.79	612
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	393	0.80	620
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	434	0.80	696
4	Heterogeneous	392	0.80	625
5	$\beta = 0.74, \tilde{\beta} = 1$	393	0.80	617
6	$\beta = 0.74, \tilde{\beta} = 0.74$	393	0.80	622
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	393	0.80	625
8	Primary, heterogeneous, learning, consume in $t = 0$	377	0.80	606
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	393	0.81	631
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	452	0.80	721

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	93.3%	82.1%	76.6%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	99.7%	93.0%	81.9%	76.0%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	98.7%	92.4%	80.0%	74.0%
4	Heterogeneous	99.3%	92.7%	81.7%	75.6%
5	$\beta = 0.74, \tilde{\beta} = 1$	99.7%	93.0%	81.9%	76.0%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	99.7%	93.0%	81.9%	76.0%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	99.7%	93.0%	81.9%	76.0%
8	Primary, heterogeneous, learning, consume in $t = 0$	93.5%	89.3%	77.0%	69.5%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	99.4%	92.8%	81.6%	75.5%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	97.3%	91.6%	77.6%	71.1%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.002$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \leq t \leq 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \geq 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A14: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0005$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	393	0.79	612
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	392	0.80	619
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	460	0.80	733
4	Heterogeneous	392	0.80	625
5	$\beta = 0.74, \tilde{\beta} = 1$	393	0.80	617
6	$\beta = 0.74, \tilde{\beta} = 0.74$	392	0.80	620
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	393	0.80	628
8	Primary, heterogeneous, learning, consume in $t = 0$	345	0.80	558
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	393	0.81	631
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	480	0.80	765

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline (\$500 cap)	(2) \$400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	91.7%	58.9%	45.4%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	99.2%	91.0%	58.4%	44.1%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	94.9%	88.6%	49.3%	33.2%
4	Heterogeneous	98.2%	90.1%	58.0%	43.3%
5	$\beta = 0.74, \tilde{\beta} = 1$	99.2%	90.9%	58.4%	43.9%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	99.2%	91.0%	58.4%	44.1%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	99.2%	91.0%	58.4%	44.0%
8	Primary, heterogeneous, learning, consume in $t = 0$	77.7%	76.6%	37.3%	18.7%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	98.3%	90.2%	57.9%	43.0%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	90.6%	86.1%	38.3%	19.8%

Notes: Panel (a) presents simulated borrowing behavior under a \$500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a \$500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0005$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \leq t \leq 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \geq 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A15: **Calibrated Using Empirical Estimates of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0002$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	396	0.79	620
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	397	0.80	633
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	422	0.81	676
4	Heterogeneous	396	0.80	630
5	$\beta = 0.74, \tilde{\beta} = 1$	396	0.80	637
6	$\beta = 0.74, \tilde{\beta} = 0.74$	397	0.79	614
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	396	0.80	627
8	Primary, heterogeneous, learning, consume in $t = 0$	351	0.80	562
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	396	0.80	631
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	433	0.81	703

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	85.6%	71.7%	65.0%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	99.5%	85.2%	71.3%	64.2%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	96.5%	83.0%	66.9%	59.4%
4	Heterogeneous	99.0%	84.7%	71.1%	63.6%
5	$\beta = 0.74, \tilde{\beta} = 1$	99.5%	85.2%	71.3%	64.1%
6	$\beta = 0.74, \tilde{\beta} = 0.74$	99.5%	85.2%	71.3%	64.2%
7	$\beta = 0.74, \tilde{\beta}_0 = 0.88, \tilde{\beta}_1 = 0.74$ (learning)	99.5%	85.2%	71.3%	64.2%
8	Primary, heterogeneous, learning, consume in $t = 0$	84.7%	75.1%	59.0%	48.6%
9	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	99.0%	84.8%	71.0%	63.5%
10	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	92.3%	79.9%	61.0%	52.4%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 0.02)$ and $\alpha_0 = 0.0002$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 8 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Rows 4 and 8 model heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in rows 2 and 7, respectively. Row 7 models learning, assuming $\beta = 0.74$, $\tilde{\beta}_0 = 0.88$ in periods $0 \leq t \leq 3$, and $\tilde{\beta}_1 = \beta$ in periods $t \geq 4$. Rows 9 and 10 set β and $\tilde{\beta}$ to match expert forecasts.

Table A16: **Calibrated Using Experts' Forecasts of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0064$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	396	0.61	516
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	394	0.75	581
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	411	0.75	605
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	395	0.81	647
5	$\beta = 0.63, \tilde{\beta} = 1$	396	0.81	648
6	$\beta = 0.63, \tilde{\beta} = 0.63$	390	0.78	602
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	421	0.81	683
8	Heterogeneous expert β	394	0.77	815

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	97.0%	99.7%	99.2%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	99.8%	96.8%	99.6%	98.9%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	99.8%	96.8%	99.5%	98.9%
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	99.4%	96.5%	99.4%	98.4%
5	$\beta = 0.63, \tilde{\beta} = 1$	99.3%	96.4%	99.4%	98.4%
6	$\beta = 0.63, \tilde{\beta} = 0.63$	99.6%	96.7%	99.4%	98.7%
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	99.3%	96.4%	99.3%	98.3%
8	Expert heterogeneous β	97.4%	94.9%	99.2%	95.8%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0064$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 7 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Row 8 models heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in row 4. Rows 4 and 7 set β and $\tilde{\beta}$ to match expert forecasts.

Table A17: **Calibrated Using Experts' Forecasts of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.002$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	397	0.61	516
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	391	0.75	577
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	426	0.75	628
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	395	0.81	646
5	$\beta = 0.63, \tilde{\beta} = 1$	397	0.82	650
6	$\beta = 0.63, \tilde{\beta} = 0.63$	384	0.78	593
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	444	0.81	722
8	Heterogeneous expert β	393	0.77	810

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	92.4%	97.2%	91.6%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	97.9%	90.8%	95.6%	89.3%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	97.2%	90.4%	94.9%	88.3%
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	93.7%	87.4%	93.9%	84.4%
5	$\beta = 0.63, \tilde{\beta} = 1$	93.2%	87.0%	93.8%	83.9%
6	$\beta = 0.63, \tilde{\beta} = 0.63$	96.1%	89.3%	94.2%	87.3%
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	91.5%	86.2%	91.9%	81.1%
8	Expert heterogeneous β	74.0%	71.0%	91.9%	59.8%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.002$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 7 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Row 8 models heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in row 4. Rows 4 and 7 set β and $\tilde{\beta}$ to match expert forecasts.

Table A18: **Calibrated Using Experts' Forecasts of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0005$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	397	0.61	517
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	393	0.75	582
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	420	0.75	621
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	396	0.81	648
5	$\beta = 0.63, \tilde{\beta} = 1$	397	0.82	658
6	$\beta = 0.63, \tilde{\beta} = 0.63$	387	0.78	594
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	436	0.81	716
8	Heterogeneous expert β	394	0.77	808

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	86.6%	96.4%	89.2%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	97.3%	84.5%	94.3%	86.2%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	96.2%	83.8%	93.1%	84.7%
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	91.9%	80.2%	92.0%	80.1%
5	$\beta = 0.63, \tilde{\beta} = 1$	91.3%	79.8%	92.0%	79.3%
6	$\beta = 0.63, \tilde{\beta} = 0.63$	95.0%	82.7%	92.5%	83.7%
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	88.3%	77.9%	88.9%	74.8%
8	Expert heterogeneous β	66.6%	59.9%	89.6%	49.2%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0005$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 7 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Row 8 models heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in row 4. Rows 4 and 7 set β and $\tilde{\beta}$ to match expert forecasts.

Table A19: **Calibrated Using Experts' Forecasts of β and $\tilde{\beta}$:** $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0002$

(a) **Simulated Borrowing Behavior**

Row	Scenario	(1) Average loan size	(2) Probability of reborrowing	(3) Average amount repaid
1	$\beta = 1, \tilde{\beta} = 1$	395	0.61	516
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	392	0.74	576
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	412	0.75	608
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	394	0.81	646
5	$\beta = 0.63, \tilde{\beta} = 1$	395	0.81	651
6	$\beta = 0.63, \tilde{\beta} = 0.63$	389	0.78	602
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	421	0.81	695
8	Heterogeneous expert β	393	0.77	801

(b) **Borrower Welfare Under Payday Lending Regulation**

Row	Scenario	(1) Baseline ($500 cap)	(2) $400 cap	(3) Rollover restriction	(4) 25% fee
1	$\beta = 1, \tilde{\beta} = 1$	100.0%	83.9%	97.3%	91.9%
2	$\beta = 0.74, \tilde{\beta} = 0.77$ (primary estimates)	98.0%	82.3%	95.8%	89.7%
3	$\beta = 0.74, \tilde{\beta} = 0.77$, consume in $t = 0$	97.2%	81.7%	94.9%	88.6%
4	$\beta = 0.63, \tilde{\beta} = 0.86$ (expert forecast)	94.0%	79.2%	94.1%	85.2%
5	$\beta = 0.63, \tilde{\beta} = 1$	93.6%	78.8%	94.0%	84.6%
6	$\beta = 0.63, \tilde{\beta} = 0.63$	96.3%	81.0%	94.4%	87.8%
7	$\beta = 0.63, \tilde{\beta} = 0.86$, consume in $t = 0$	91.0%	77.1%	91.6%	81.3%
8	Expert heterogeneous β	75.4%	64.4%	92.3%	62.1%

Notes: Panel (a) presents simulated borrowing behavior under a $500 loan size cap. Panel (b) presents welfare estimates as a percent of the surplus that time-consistent borrowers derive from the availability of payday loans under a $500 loan size cap. Both panels assume that $\theta \sim Beta(a_\theta, 1)$ and $\alpha_0 = 0.0002$. In Panel (b), "Rollover restriction" in column 3 refers to the requirement that borrowers repay by period $t = 3$ at the latest. "25% fee" in column 4 refers to an increase in the borrowing fee from 15% to 25%, which might be caused by substitution to higher-cost credit after a payday loan ban. Rows 3 and 7 present alternative analyses where the benefits of the loan accrue fully in $t = 0$, so borrowers overborrow relative to the welfare criterion. Row 8 models heterogeneity, where half the population is time-consistent and the other half has β and $\tilde{\beta}$ such that the population averages correspond to the assumptions in row 4. Rows 4 and 7 set β and $\tilde{\beta}$ to match expert forecasts.

I Borrower Survey Screenshots

Figure A16: **Introduction and Consent**

Introduction

This survey is part of a research collaboration between [Lender] and researchers at a group of universities: Stanford, Berkeley, NYU, and Dartmouth. Our goal is to learn more about payday loan customers.

The survey should take about 5-10 minutes. To thank you for your time, you will receive a $10 cash card after you finish all of the questions. You will also have the opportunity to earn an additional cash reward up to $160. About 40% of participants will be offered an additional reward. *Note that if you took the survey in 2018, you are eligible to take the survey again. You can only take the survey once in 2019. To participate, you must have taken out a payday loan from [Lender] in Indiana in the past 30 days. You will only be paid once for completing the survey.*

If you participate in this survey, the researchers will analyze data regarding your borrowing history provided by [Lender] and third-party data sources such as Veritec Solutions, LLC and/or Clarity Services. By participating in this survey, you permit Veritec Solutions and/or Clarity Services to share data, including data obtained from lenders, with the parties and third parties involved in this research project, including the company issuing payments for the research awards.

If you participate in the survey, your data will be confidential and will be used for research purposes only. Your individual answers will not be given to [Lender] and will not impact your ability to borrow from [Lender] or other lenders. De-identified data from this research project may be made publicly available for replication studies. All identifying information will be removed and we will maintain your privacy in all published and written data resulting from this study.

If you have any questions, you can contact the research team at mhorste@poverty-action.org or at PaydayResearch_Kim@stanford.edu. If you have any concerns or complaints about the research, please contact the Stanford Institutional Review Board at (650) 723-2480 or via email at IRB2-Manager@lists.stanford.edu. You can also write to the Innovations for Poverty Action Institutional Review Board at humansubjects@poverty-action.org.

Given the above information, do you wish to participate in the survey?

○ I want to participate! I hereby certify that by clicking this box and participating in the research survey, I have read and understood the information described above. Furthermore, I consent to and authorize the researchers to obtain information and data about me from the data sources listed above.

Figure A17: **Personal Information**

INTRODUCTION

Before we begin, we'd like to get a bit of background information. **Please answer carefully. If we can't match your information to your [Lender] borrowing records, we won't be able to process your survey rewards.**

First Name

Last Name

Date of birth (mm/dd/yyyy)

Email address

Figure A18: **Predictions about Future Borrowing**

FUTURE BORROWING

First, we'd like to ask your opinion about how likely you are to get another payday loan from any lender before **[8 weeks from now].**

We'd like you to give us a number from 0 to 100, where 0 means there is absolutely no chance and 100 means it's absolutely sure to happen.

For example, no one can ever be sure about tomorrow's weather, but if you think that rain is very unlikely tomorrow, you might say that there is 10% chance of rain. If you think that there is a very good chance that it will rain tomorrow, you might say that there is 80% chance of rain.

What do you think is the chance that you will get another payday loan from any lender before **[8 weeks from now]?**

	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Choose one	○	○	○	○	○	○	○	○	○	○	○

Figure A19: **"$100 If You Are Debt-Free" Description**

FUTURE BORROWING

Imagine the computer selects you for the **$100 If You Are Debt-Free** reward. This would give you an incentive to avoid getting another payday loan. We would like to learn how much you think it would reduce your chance of getting another payday loan.

If you are selected for $100 If You Are Debt-Free, what is the chance that you would get another payday loan from any lender before [8 weeks from now]?

	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Choose one	○	○	○	○	○	○	○	○	○	○	○

Reminders:

- Earlier, you told us that your chance of getting a payday loan before [8 weeks from now] WITHOUT **$100 If You Are Debt-Free** was 80%.
- If the computer selects you for **$100 If You Are Debt-Free**, we will send you $100 if you **do not** get another payday loan from [Lender] or *any other payday lender* before [8 weeks from now]. We would send you the money by [12 weeks from now] on a Visa cash card.
- Your answer to this question won't affect your chance of being selected for **$100 If You Are Debt-Free**, and we won't give this or your other answers to [Lender].

Figure A20: **Predictions about Future Borrowing with Incentive**

$100 IF YOU ARE DEBT-FREE

If you complete this survey, the computer may select you for an additional reward. The first possible reward is **$100 If You Are Debt-Free**.

If the computer selects you for **$100 If You Are Debt-Free**, we will send you $100 if you **do not** get another payday loan from [Lender] *or any other payday lender* before *[8 weeks from now].** We would send you the money by **[12 weeks from now]** on a Visa cash card.

Note: All payday lenders are required to report loans to a database. We will use that database to check your borrowing from all payday lenders.

We want to make sure we explained this clearly. Which of the following is true?

If the computer selects me for **$100 If You Are Debt-Free,** then:

- ○ If I don't get another payday loan from any lender before [8 weeks from now], I will receive a $100 Visa cash card by [12 weeks from now]
- ○ If I get another payday loan before [8 weeks from now] from Advance America, I will NOT receive the $100 Visa cash card.
- ○ If the database shows that I got another payday loan before [8 weeks from now] from another payday lender, I will NOT receive the $100 Visa cash card
- ○ All of the above

Figure A21: **"Money for Sure" Description**

WHICH REWARD DO YOU PREFER?

The second possible reward for completing this survey is simply **Money for Sure.** It is paid the same way as the **$100 If You Are Debt-Free**: we would send you the money by [12 weeks from now] on a Visa cash card.

Money For Sure is exactly what it sounds like: You get it for sure, REGARDLESS of whether or not you get another payday loan.

Figure A22: **Introduction to the Multiple Price List**

WHICH REWARD DO YOU PREFER?

Now you get to tell us how you would choose between **Money For Sure** and **$100 If You Are Debt-Free.**

Think carefully, because the computer may randomly select one of the following questions and give you what you chose in that question. Click here if you want more information on how the computer randomly selects questions.

Figure A23: **MPL Example 1**

How might you decide?

Earlier, you told us that you have a **40%** chance of getting another payday loan before [8 weeks from now] if you are selected for **$100 If You Are Debt-Free**. In other words, you would have a **60%** chance of being debt-free. So on average, **$100 If You Are Debt-Free** would earn you $60**.**

Given that, which reward would you prefer?

- ○ **$60 For Sure**. This gives you certainty and avoids pressure to stay debt-free.
- ○ **$100 If You Are Debt-Free**. This gives you extra motivation to stay debt-free.

Figure A24: **MPL Example 2**

WHICH REWARD DO YOU PREFER?

Now we are going to ask you a similar question, but for a different amount of **Money For Sure** that the computer has selected.

Which reward would you prefer?

- ○ **$100 if You Are Debt-Free**. Given your chance of getting another payday loan, on average this earns you $20 less in rewards. This also gives you extra motivation to stay debt-free.
- ○ **$80 For Sure**. Given your chance of getting another payday loan, on average this earns you $20 more in rewards. This also gives you certainty and avoids pressure to stay debt-free.

Figure A25: **MPL Example 3**

WHICH REWARD DO YOU PREFER?

Now we are going to ask you a similar question, but for a different amount of **Money For Sure** that the computer has selected.

Which reward would you prefer?

- ○ **$100 if You Are Debt-Free**. Given your chance of getting another payday loan, on average this earns you $20 more in rewards. This also gives you extra motivation to stay debt-free.
- ○ **$40 For Sure**. Given your chance of getting another payday loan, on average this earns you $20 less in rewards. This also gives you certainty and avoids pressure to stay debt-free.

Figure A26: **“Flip a Coin for $100” Description**

FLIP A COIN FOR $100

The third and final possible reward for completing this survey is **Flip a Coin for $100**. It is paid the same way as the other two rewards: we would send you the money by [12 weeks from now] on a Visa cash card.

If you're selected for **Flip a Coin for $100**, the computer will flip a (computerized) coin. You'll have a 50% chance of winning $100 and a 50% chance of winning nothing. So on average, **Flip a Coin for $100** would earn you $50.

Figure A27: **Introduction to Flip a Coin MPL**

FLIP A COIN FOR $100

Now you get to tell us how you would choose between **Money For Sure** and **Flip a Coin for $100.**

Think carefully, because the computer may randomly select one of the following questions and give you what you chose in that question. Click here if you want more information on how the computer randomly selects questions.

Figure A28: **Flip a Coin MPL Example 1**

FLIP A COIN FOR $100

Which reward would you prefer?

- ○ **$50 For Sure**
- ○ **Flip a Coin for $100**

Figure A29: **Flip a Coin MPL Example 2**

WHICH REWARD DO YOU PREFER?

Now we are going to ask you a similar question, but for a different amount of **Money For Sure**.

Which reward would you prefer?

- ◯ **$70 For Sure**
- ◯ **Flip a Coin for $100**

Figure A30: **Flip a Coin MPL Example 3**

WHICH REWARD DO YOU PREFER?

Now we are going to ask you a similar question, but for a different amount of **Money For Sure**.

Which reward would you prefer?

- ◯ **Flip a Coin for $100**
- ◯ **$30 For Sure**

Figure A31: **Final Questions**

FINAL QUESTIONS

We have three final questions.

To what extent would you like to give yourself extra motivation to avoid payday loan debt in the future?

- ◯ Very much
- ◯ Somewhat
- ◯ Not at all

In the past, how has your expected payday loan usage lined up with reality?

- ◯ I usually ended up getting payday loans less often than I expected
- ◯ I usually ended up getting payday loans about as often as I expected
- ◯ I usually ended up getting payday loans more often than I expected

Some states have laws that prohibit people from taking out payday loans more than three paydays in a row. Do you think such a law would be good or bad for you?

- ◯ Very good
- ◯ Somewhat good
- ◯ Neutral
- ◯ Somewhat bad
- ◯ Very bad

J Expert Survey Screenshots

Figure A32: **Introduction**

Consent Form

This survey is a part of a study by Hunt Allcott (NYU), Joshua Kim (Stanford), Dmitry Taubinsky (UC Berkeley), and Jonathan Zinman (Dartmouth). The goal of the survey is to get a sense of what academics' prior beliefs are about whether payday loan regulation may be welfare enhancing, and the degree to which borrowers may or may not be making mistakes. The survey should take about 3 minutes. Participation is voluntary and your data will be used for research purposes only.

We will be happy to email you the results of this survey and a copy of our completed manuscript.

This survey is part of a study by Hunt Allcott (NYU), Josh Kim (Stanford), Dmitry Taubinsky (UC Berkeley) and Jonathan Zinman (Dartmouth). Feel free to email Dmitry with any questions or concerns (dmitry.taubinsky@berkeley.edu). If you have any concerns of complaints about the research, please contact the Stanford Institutional Review Board at (650) 723-2480 or via email at IRB2-Manager@lists.stanford.edu

Given the above information, do you wish to participate in the survey?

- Yes, I wish to participate
- No, I do not wish to participate

Figure A33: **Background Information**

Do you have a PhD in economics?

- ○ Yes
- ○ No

Which best describes your primary employer?

- ○ US Congress
- ○ State regulatory agency
- ○ State legislature
- ○ Federal agency (CFPB, Federal Reserve, FCA, etc.)
- ○ Payday lender or other financial services company
- ○ Think tank or advocacy organization
- ○ University
- ○ Other (please explain)

Figure A34: **Market Background**

Market background

As you are probably well aware, payday loans are short-term loans designed to be fully repaid on or soon after the borrower's next payday. In data we are studying, the average loan size is $373, the modal loan maturity is 14 days, the interest rate is $10-$15 per $100 borrowed (depending on loan amount), and full repayment (principal + interest) is due in a single payment at maturity.

Borrowers often "re-borrow", either by paying late (incurring additional interest and fees) or by fully repaying but then taking out a new loan soon after. (Renewals/refinancing/rollovers are prohibited by law in our study setting.)

Figure A35: **Opinions about Payday Loan Bans**

Some states have laws that effectively prohibit payday lending, for example by imposing a low interest rate cap. Do you think such a law is good or bad for consumers overall?

- ○ Very good
- ○ Somewhat good
- ○ Neutral
- ○ Somewhat bad
- ○ Very bad

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○

Figure A36: **Opinions about Rollover Restrictions**

Imagine a law that successfully enforces a one-month "cooling off period" for any individual who takes out payday loans more than three paydays in a row or for an individual who takes out but does not repay a payday loan. During the cooling off period, the individual could not borrow from any payday lender. Do you think such a law is good or bad for consumers overall?

- ○ Very good
- ○ Somewhat good
- ○ Neutral
- ○ Somewhat bad
- ○ Very bad

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○

Figure A37: **Opinions about Loan Size Caps**

Pew Charitable Trusts has proposed a law that effectively limits payday loan amounts to no more than 5% of the borrower's expected gross income over the loan repayment period. Do you think such a law is good or bad for consumers overall?

- ○ Very good
- ○ Somewhat good
- ○ Neutral
- ○ Somewhat bad
- ○ Very bad

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○

Figure A38: **Opinions about Borrower Decision-Making**

Opinions about borrower decision-making

Finally, we'd like to ask for your predictions about key aspects of borrower behavior in our study. From January 7th to March 29th, 2019, we surveyed 1,205 payday borrowers from one of the USA's largest payday lenders in several of the lender's stores in Indiana. Indiana state law prohibits renewals/refinancing/rollovers but has only mild restrictions on re-borrowing consecutively or from multiple lenders. We then tracked subsequent borrower behavior with data from our partner lender and the Veritec small-dollar loan database used to track compliance with state regulations.*

Do you think that the average payday loan borrower in our sample correctly foresees the chance that she will re-borrow in the next 60 days?

Remember: *re-borrowing* is defined as (a) paying late (incurring additional fees) or (b) fully repaying but then taking out a new loan soon after from our lender or any other lender that reports into the Indiana Veritec database.

- ○ Yes
- ○ No, I think the average borrower overestimates the chance she will re-borrow in the next 60 days.
- ○ No, I think the average borrower underestimates the chance she will re-borrow in the next 60 days.

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○

Figure A39: **Beliefs about Borrowers' Predicted Reborrowing Probability**

Opinions about borrower decision-making

In data we've been analyzing, the average payday loan borrower has about a 70% chance of re-borrowing within the next 60 days. Above, you answered that you think the average person underestimates that probability. What do you think the average borrower in our data *believes* is that probability? (*Please answer in percentage points, from 0 to 100.*)

Figure A40: **Beliefs about Borrowers' Demand for Motivation**

Do you think that the average payday loan borrower would want to give herself extra motivation to avoid re-borrowing in the future? (In technical terms, do you think that the average borrower is present-biased / time inconsistent / has costly self-control and is at least partially sophisticated about it?)

- Yes
- No

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○

Figure A41: **Beliefs about $\tilde{\beta}$**

What do you think is the average beta-hat (perceived present bias parameter in a beta-delta-beta-hat model) of payday borrowers? If you are not familiar with the model, please write N/A.

Figure A42: **Beliefs about Whether Borrowers Say They Want Motivation**

We also asked borrowers, “Would you like to give yourself extra motivation to avoid payday loan debt?” The possible answers were “not at all,” “somewhat,” and “very much.”

What percent of borrowers do you think answered “very much”? *(Please give an answer from 0 to 100.)*

How certain are you of your answer?

	Not certain at all	Slightly certain	Moderately certain	Very certain	Extremely certain
Choose one	○	○	○	○	○